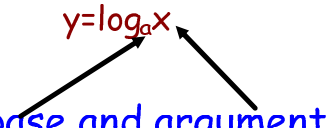
5.6 Solving Logarithmic Equations

Strategies:

- rewrite decimals/roots as fractions
- change form log⇒exp
- use log laws to combine logs
- get LS and RS expressed as a single log with the same base

Warnings:

- remember that both the base and argument of a log must be positive



- watch for extraneous roots

A root that is not a possible solution.

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Ex 1 State the restrictions on x for the following.

a) $y=\log_x 8$

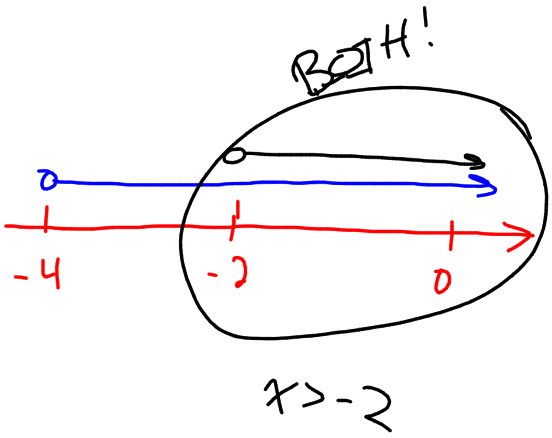
$\{x \in \mathbb{R} \mid x > 0\}$

b) $\log_5(x+4) - \log_5(x+2) = 7$

$x+4 > 0$ $\quad$ $x+2 > 0$

$x > -4$ $\quad$ $x > -2$

$x > -2$



$x > -2$

Ex. 2 Solve the following. Give exact values where possible, otherwise round to 2 decimal places.

a) $\log_x 0.0625 = -4$

$\log_x \frac{1}{16} = -4$

$x^{-4} = \frac{1}{16}$

$(x^{-4})^{-\frac{1}{4}} = (\frac{1}{16})^{-\frac{1}{4}}$

$x^{-1} = \frac{1}{2}$

$x = 2$

$x^{-1} = 2^{-1}$

$\therefore x = 2$

$\frac{1}{x} = \frac{1}{2}$

$\therefore x = 2$

$0.0625 = \frac{625}{10000} = \frac{1}{16}$

b) $\log_5 (2x-4) = \log_5 36$

Since we have $\log_5$ on both side, we can cancel

$2x - 4 = 36$

$2x = 40$

$x = 20$ ✓

Restrictions?

$2x - 4 > 0$

$2x > 4$

$x > 2$

c) $\log_3 12 - \log_3 x = \log_3 3$

$\log_3 (\frac{12}{x}) = \log_3 3$ ← using Log laws

$\frac{12}{x} = 3$

$x = 4$ ✓

Restrictions!

$x > 0$

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d) $3\log x + 2\log x = 20$

$\log_{10} x^3 + \log_{10} x^2 = 20$

$\log_{10}(x^3 \cdot x^2) = 20$

$\log_{10} x^5 = 20$

$5\log_{10} x = 20$

$\log_{10} x = 4$

$10^4 = x$

$\therefore x = 10000$ ✓

Restrictions!

$x > 0$

e) $\log_3(2x+3) + \log_3 x = 2$

$\log_3[(2x+3)(x)] = 2$

$3^2 = (2x+3)(x)$

$9 = 2x^2 + 3x$

$0 = 2x^2 + 3x - 9$

$= (x+3)(2x-3)$

Restrictions!

$2x+3>0 \quad x>0$

$2x > -3$

$x > -\frac{3}{2}$

$x > 0$

$x = -3$ Inadmissable

$x = \frac{3}{2}$ ✓

M - 18

A 3

N $\frac{2}{6}$ $\frac{?}{-3}$

$\frac{1}{3}$ $\frac{2}{-3}$

f) $\log_6(x+3) + \log_6(x-2) = 1$

$\log_6[(x+3)(x-2)] = 1$

Restrictions!

$x+3>0 \quad x-2>0$

$x > -3 \quad x > 2$

$x > 2$

$6^1 = (x+3)(x-2)$

$0 = x^2 + x - 6 - 6$

$= x^2 + x - 12$

$= (x+4)(x-3)$

$x = -4$ Inadmissable

$x = 3$

$\therefore x = 3$

$\log_6[(x+3)(x-2)] = \log_6 6$

$(x+3)(x-2) = 6$

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Ex 3 Solve the following in two different ways.

$\log_2 x - \log_2 3 = \log_2 6$

$\log_2\left(\frac{x}{3}\right) = \log_2 6$

$\frac{x}{3} = 6$

$x = 18$

$\log_2 x = \log_2 3 + \log_2 6$

$\log_2 x = \log_2 18$

$x = 18$

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HOMEWORK

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