5.7 Solving Problems with Exponential and Logarithmic Functions

Logarithmic Scales:

When quantities vary over very large ranges it is sometimes convenient to take their logarithm in order to get a more manageable set of numbers. With logarithms, each increase of 1 unit corresponds to a multiplication by a factor of 10.

A. The Richter Scale (base 10)

Magnitude, M:

$M = \log\left(\frac{I}{I_0}\right)$ where $\frac{I}{I_0}$ is the ratio of intensities between the earthquake being measured and that of a standard low-level earthquake.

Richter Scale Magnitude	True Intensity
$1 = \log_{10}10^1$	$10^1 = 10$
$2 = \log_{10}10^2$	$10^2 = 100$
$3 = \log_{10}10^3$	$10^3 = 1000$
$4 = \log_{10}10^4$	$10^4 = 10\ 000$

$$\frac{10^4}{10^2} = 10^2$$

higher

$$\text{ratio} = \frac{10^{\square}}{10^{\square}}$$

← lower

A magnitude of 4 compared to a magnitude of 2 is 100 times more intense.

Ex 1 How many times more intense was the 1964 Alaska earthquake of magnitude 8.5 compared to an earthquake of magnitude 6.0?

$$\text{ratio} = \frac{10^{8.5}}{10^6}$$

$$= 10^{2.5} \rightarrow \doteq 316.2$$

∴ The 8.5 earthquake was approx. 316.2 times greater

Ex 2 Sarah claims that she was in an earthquake that was 125 x as intense as one measuring 5.2 on the Richter scale. What was the magnitude of the earthquake she was in?

∴ Let x be the greater magnitude

$$\text{ratio} = \frac{10^{\square}}{10^{\square}}$$

$$125 = \frac{10^x}{10^{5.2}}$$

$$10^x = 125 \cdot 10^{5.2}$$

$$x \cdot \log_{10} 10 = \log_{10}(125 \cdot 10^{5.2})$$

$$x = \frac{\log_{10}(125 \cdot 10^{5.2})}{\log_{10} 10}$$

$$\doteq 7.3$$

∴ The magnitude of her quake was 7.3

B. The Decibel Scale (base unit is the "Bel"..measures loudness)

$$L = 10 \log \left(\frac{I}{I_o} \right)$$

intensity of a given sound [watts/m²]

Loudness (dB)

intensity of a barely audible sound

$I_o = 10^{-12}$ w/m²

$$\text{ratio} = \frac{10^{\square}}{10^{\square}} \quad (\text{in B})$$

higher intensity

Intensity (I)	Ratio (I / Io)	Loudness (B)	Loudness (dB)
10^{-12}	10^{0}	0	0
10^{-11}	10^{1}	1	10
10^{-10}	10^{2}	2	20
10^{-9}	10^{3}	3	30
10^{-8}	10^{4}	4	40
10^{-7}	10^{5}	5	50

Ex 3 How many times louder is a stereo measuring 97 dB than a conversation measuring 43 dB?

$$\text{ratio} = \frac{10^{\square}}{10^{\square}}$$

$$= \frac{10^{9.7}}{10^{4.3}}$$

97 dB

= 9.7 B

$$= 10^{5.4}$$

$$\doteq 251\,188.6$$

∴ It is approx. 251 188.6 times louder

C. The pH Scale

- measure of acidity or alkalinity
- # of hydrogen ions (H+) in a mole of substance
- 1 mole = 6×10^{23} particles
- scale ranges from 0 to 14
- 0 ⇒7 is acidic, 7 is neutral, 7⇒14 is basic

Formula: $pH = -\log[\text{concentration of } H^+]$

or $pH = -\log[H^+]$ ← measured in mol/L

Ex 4 Determine the pH, where the H^+ concentration is 0.0047 mol/L.

$pH = -\log_{10}[H^+]$

$pH = -\log_{10}(0.0047)$

$\doteq 2.3$

∴ the pH is 2.3

Ex 6 Calculate the # of moles of H^+ per litre for a solution with pH = 7.4

→ concentration

$pH = -\log_{10}[H^+]$

$7.4 = -\log_{10}[H^+]$

$-7.4 = \log_{10}[H^+]$

$10^{-7.4} = [H^+]$

$[H^+] \doteq 4.0 \times 10^{-8}$

∴ the $[H^+]$ is 4.0×10^{-8} mol/L

D. Exponential Growth and Decay

the initial amount

the number of growth/decay periods

Formula: $y = c\ (a^x)$

the final amount

the growth/decay factor
(1 + growth rate or 1 - decay rate)

aka $A = a_0 b^x$

Ex 5 A $150 000 yacht depreciates 10% per year. How long does it take for the yacht to be worth $96 000?

$A = 96000$

$a_0 = 150\ 000$

$b = 1 - 0.1$

$= 0.9$

$x = ?$

$A = a_0 b^x$

$96000 = 150\ 000(0.9)^x$

$\frac{16}{25} = (0.9)^x$

$\log_{10}(\frac{16}{25}) = x \log(0.9)$

$\log_{0.9}\frac{16}{25} = x$

$x = \frac{\log_{10}\frac{16}{25}}{\log_{10}0.9}$

$\doteq 4.2$

4 years
and .2 × 12
months
≈ $2\frac{1}{2}$ months

∴ It will take 4 years and 2 months

Extra Review

p. 477, 478

p. 509

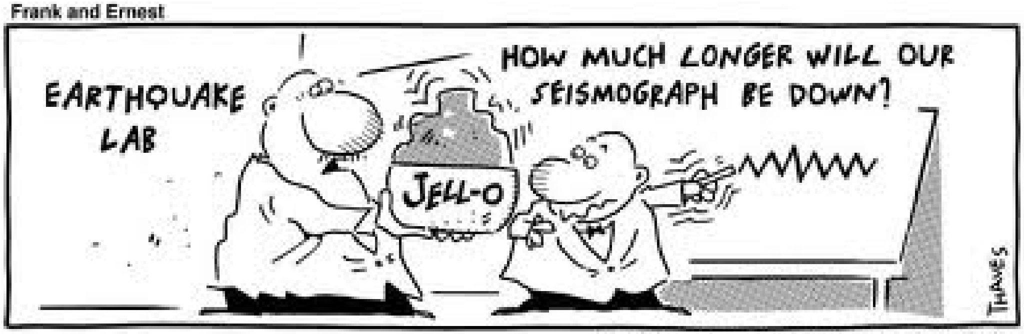
p. 510 #1-22

p. 512 #1-9 (not 8d)

Homework

p. 499 #1-3,8,10,13-15

Frank and Ernest



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