6.1 Average Rate of Change (A.R.o.C.)

Definition:

The average rate of change of a relation describes how the dependent variable changes(Δy) with respect to the independent variable(Δx) over a specific interval in the domain.

Algebraically:

SLOPE!

average rate of change=

➡ $\frac{\text{change in dependent variable}}{\text{change in independent variable}}$

➡ $\frac{\Delta y}{\Delta x}$

➡ $\frac{y_2 - y_1}{x_2 - x_1}$

➡ $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$

Pull

A.R.o.C. from a table.

Ex. 1 The table shows the temperature of an oven as it heats up from room temperature to 400°F.

Time t (min)	Temp (°F) T
0	70
1	125
2	170
3	210
4	250
5	280
6	310
7	335
8	360
9	380
10	400

a) Determine the average rate of change for each interval.

i) $0 \le t \le 3$

$ARoC = \frac{\Delta T}{\Delta t}$

$= \frac{210-70}{3-0}$

$\doteq 46.7$ °F/min

ii) $3 \le t \le 6$

$ARoC = \frac{\Delta T}{\Delta t}$

$= \frac{310-210}{6-3}$

$\doteq 33.3$ °F/min

iii) $6 \le t \le 9$

$ARoC = \frac{\Delta T}{\Delta t}$

$= \frac{380-310}{9-6}$

$\doteq 23.3$ °F/min

b) Discuss/ explain what the answers to a, b, and c tell you about how the oven temperature is changing.

- Increasing (+'ve #), therefore getting hotter
- Oven heats much faster initially

c) How would these values change if the oven was cooling?

- Would decrease steeply initially

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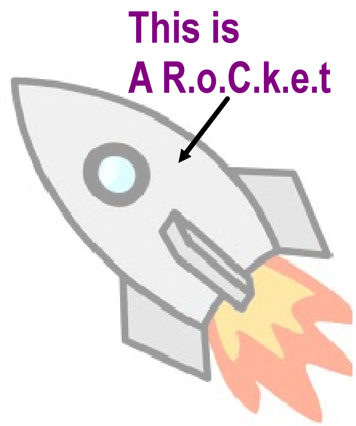
A.R.o.C. from an Equation

Ex 2 The height (m) of a model rocket t seconds after being launched is modelled by $h(t)= -4.9t^2 + 25t + 2$. Determine the average rate of change for the given interval.



a) $0 \le t \le 2$

$$ARoC = \frac{h(2)-h(0)}{2-0} = \frac{32.4-2}{2} = 15.2\ m/s$$

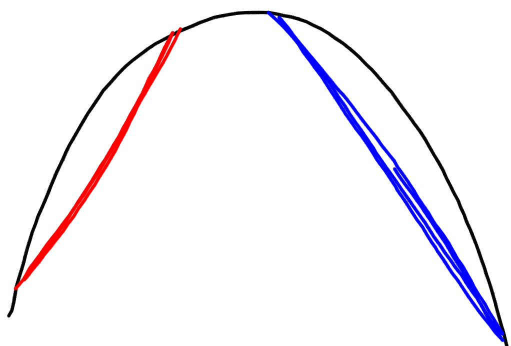
$$h(2) = -4.9(2)^2 + 25(2) + 2 = 32.4$$

$$h(0) = 2$$

b) $3 \le t \le 5$

$$ARoC = \frac{h(5)-h(3)}{5-3} = \frac{4.5-32.9}{2} = -14.2\ m/s$$

c) Explain what the values above mean in this situation.



- Slope is +'ve
- height increasing

$0 \le t \le 2$

- Slope -'ve
- Height decreasing

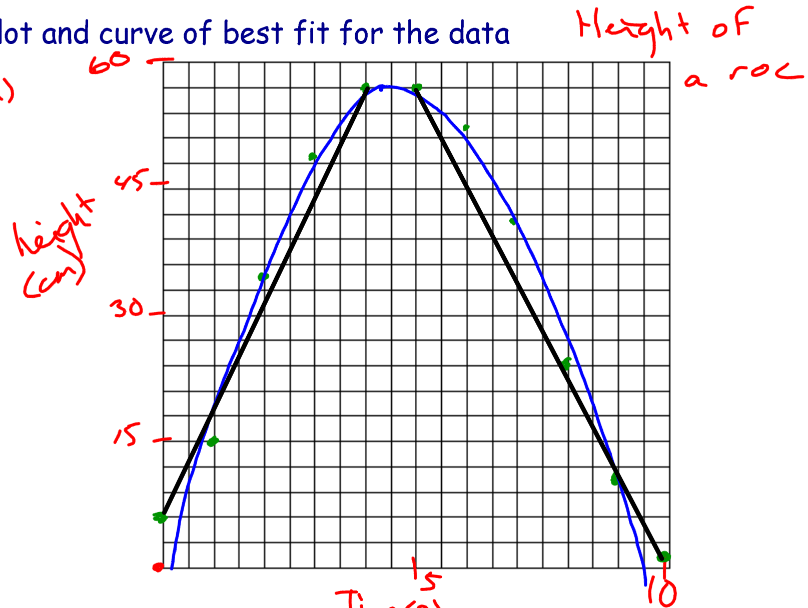
$3 \le t \le 5$

A.R.o.C. from a Graph

Ex 3 The table below shows the height of a. r.o.c. after being thrown into the air.

a) Create a scatter plot and curve of best fit for the data

Time(s)	Height (cm)
0	2
1	15
2	35
3	49
4	57
5	57
6	52
7	41
8	24
9	10
10	1.8



b) Draw a secant line from t=0 to t=4 and another from t=5 to t=10.

a line that passes through 2 points on the graph

c) Determine the average rate of change by using the endpoints of each of the secant lines in b).

$$m_{secant} = \frac{rise}{run} = \frac{57-2}{4-0} = 13.8\ \frac{cm}{s}$$

$$m_{secant} = \frac{rise}{run} = \frac{1.8-57}{10-5} = -11.0\ cm/s$$

The average rate of change from x_1 to x_2 IS the slope of the secant line from x_1 to x_2.

$$ARoC = m_{secant}$$

A.R.o.C's and Linear Functions slope!

Ex 4 Given the function f(x)=3x-4, determine the average rate of change for each interval.

a) $-2 \le x \le 1$

$$ARoC = \frac{f(1)-f(-2)}{1-(-2)}$$
$$= \frac{-1-(-10)}{3}$$
$$= \frac{9}{3}$$
$$= 3$$

$$ARoC = 3$$

b) $7 \le x \le 9$

$$ARoC = \frac{f(9)-f(7)}{9-7}$$
$$= \frac{23-17}{2}$$
$$= 3$$

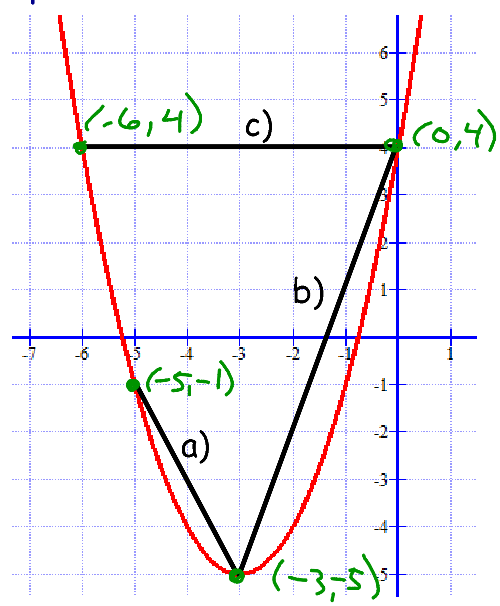
Note: The average rate of change of linear function is a constant equal to the slope of the line.

Ex 5 Given the function $f(x)=(x+3)^2-5$ determine the average rate of change for the secants drawn on the graph.

a) $m_{SECANT} = \frac{rise}{run}$
$= \frac{-4}{2}$
$= -2$

b) $m_{SECANT} = \frac{rise}{run}$
$= \frac{9}{3}$
$= 3$

c) $m_{SECANT} = \frac{rise}{run}$
$= 0$



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