6.3 Estimating Instantaneous Rates of Change (IRoC)

The height (m) of a model rocket is given by $h(t) = -4.9t^2 + 25t + 2$ over time in seconds. The table displays the average rate of change of height over an increasing smaller interval of time.

a) Complete the table (take 5-10 minutes to complete)

b) Predict what will happen to the average rate of change as the interval continues to get smaller.

c) Describe what happens to the slope of the secant as the interval gets smaller.

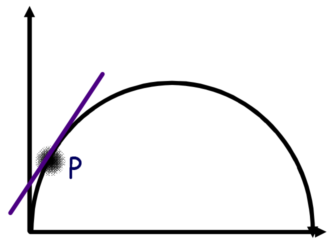
$h = -4.9t^2 + 25t + 2$

Interval	$\Delta h = h_2 - h_1$	$\Delta t = t_2 - t_1$	$ARoC = \frac{\Delta h}{\Delta t}$
$1 \le t \le 2$	$h(2) - h(1) = 10.3$	$2 - 1 = 1$	$\frac{10.3}{1} = 10.3$
$1 \le t \le 1.5$	$h(1.5) - h(1) = 6.375$	$1.5 - 1 = 0.5$	$\frac{6.375}{0.5} = 12.75$
$1 \le t \le 1.1$	$h(1.1) - h(1) = 1.471$	$1.1 - 1 = 0.1$	$\frac{1.471}{0.1} = 14.71$
$1 \le t \le 1.01$		0.01	15.151
$1 \le t \le 1.001$	0.015951	0.001	15.19551
$1 \le t \le 1.0001$	0.00151995	0.0001	15.19951
$1 \le t \le 1.00001$	0.00015995	0.00001	15.19995

As the time intervals decrease, the average rate of change(which corresponds to the slope of a secant line), becomes closer to (or approaches) 15.2 m/s. The velocity of the rocket after 1 seconds is approximately 15.2 m/s

Definition: Instantaneous Rate of Change

The instantaneous rate of change is represented graphically by the slope of a tangent to the function at a specific value of the independent variable.



A tangent line is a line that touches a curve at one point, P.

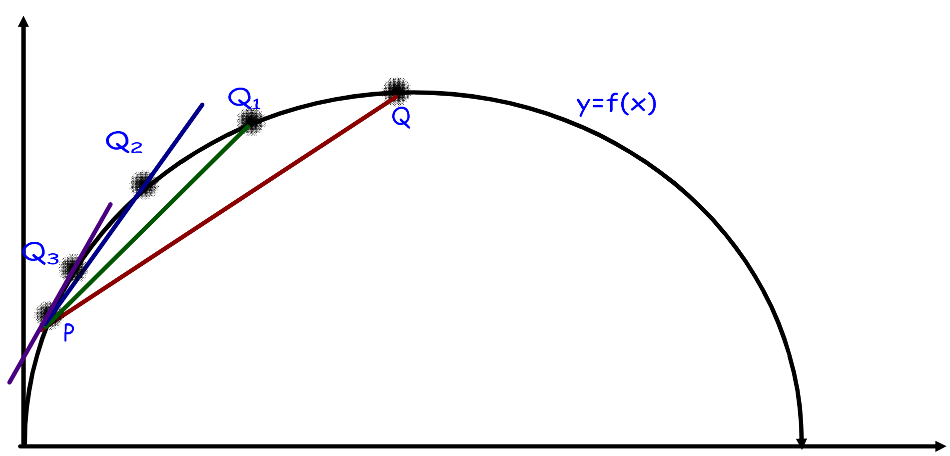
Instantaneous Rate of Change = Slope of Tangent

How do you find the slope of a tangent if, by definition, you only have one point on the line?

This is traditionally known as the "Tangent Problem" and is the basis for Calculus.

since you need two points to calculate the slope, calculate the slope of secant PQ. Allow the point Q to approach P, to make the "x" intervals smaller and smaller.

GSP



As the point Q moves toward point P, the secant line becomes the tangent line at point P

An approximate value for an instantaneous rate of change at a point can be determine many ways:

1) Preceding and Following Interval -take a value *very close* on either side of the specific value then take the average of the two ARoC

2) Difference Quotient (we will see this later in the unit)

3) Centered Intervals (covered in the textbook)

4) Slope of Tangent -calculating the slope of the tangent using rise over run on a grid.

Using Preceding and Following Intervals

Ex. 2 The distance (km) an airplane has flown after t hours is given by $s(t)=600t+30t^2-4t^3$. Estimate the instantaneous rate of change of distance at exactly 2 hours. Use 2 very small intervals on each side of t=2.

(velocity anyone??)

$s(t)=600t+30t^2-4t^3$

	Interval	ΔS	Δt	ARoC
Right	$2 \le t \le 2.0001$	$s(2.0001)-s(2)$ $= 0.0672006$	0.0001	672.0006
Left	$1.9999 \le t \le 2$	$s(2)-s(1.9999)$ $= 0.0672$	0.0001	671.9994

$\therefore$ The RoC is 672 km/h @ t = 2 hours

Sep 14-10:48 AM

Ex 3: The height, h, in metres, of a car above the ground as a Ferris Wheel turns can be modelled using the function

$h = 20\sin\left(\frac{\pi t}{60}\right) + 2.5$ where t is the time in seconds.

Estimate a value for the instantaneous rate of change of h at t=10s.

	Interval	Δh	Δt	ARoC
Right	$10 \le t \le 10.0001$	$s(10.0001) - s(10)$ $= 0.00009069$	0.0001	0.9069
Left	$9.9999 \le t \le 10$	$s(10) - s(9.9999)$ $= 0.00009069$	0.0001	0.9069

$\therefore$ The IRoC is 0.9069 m/s @ $t = 10$s

Just for fun: Link to interactive guy...surfing a curve....

http://www.ies.co.jp/math/java/calc/doukan/doukan.html

Summary:

- average rate of change = slope of secant
- instantaneous rate of change= slope of tangent

Nov 27-10:16 PM

Homework:

p. 85 #1,3,4a,9,10

p. 304 #5b)

p. 370 #2

p. 507 #6

Sep 14-12:21 PM