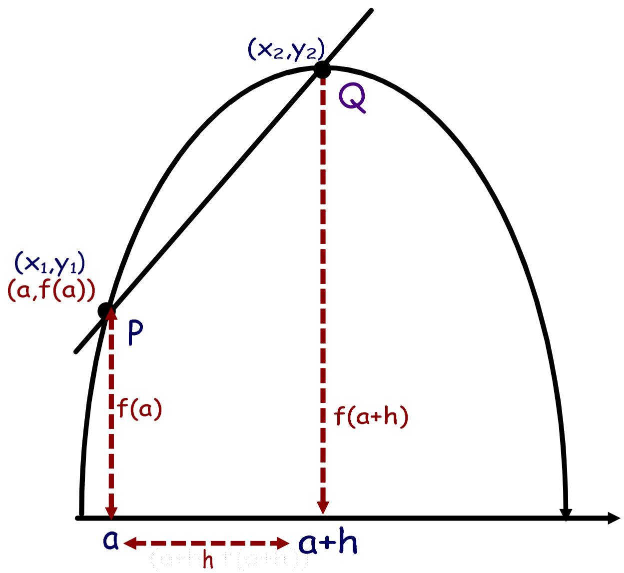
6.5 Slope of The Tangent

Method 2: Using a Difference Quotient

- In method 1 we used the slope of the secant to estimate the slope of a tangent. Formally written, for the function f(x), we were using two points:



- P (a, f(a))

  Q (a+h,f(a+h))

  where h was an increasingly small value.

- IRoC $= \dfrac{\Delta y}{\Delta x}$

$$= \frac{f(a+h)-f(a)}{(a+h)-a}$$

$$= \frac{f(a+h)-f(a)}{h}$$

Where "a" is the value of the independent value (x) at the point of tangency and h is a very, very, very small number

- The slope of this teeny, tiny secant can be found using Δy and Δx from P to Q. The resulting formula is called the DIFFERENCE QUOTIENT.

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Ex 1:

From a 10 m platform tower, a diver performs a handstand dive. Her height (m) above the water at t seconds is modelled by $d(t) = 10 - 4.9t^2$. Use the difference quotient to estimate the rate of change of height when t= 1s.

$a =$ (point of tangency) $h = 0.0001$ $a+h = 1.0001$
$= 1$

$$IRoC = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{f(1.0001) - f(1)}{0.0001}$$

$$= \frac{[10 - 4.9(1.0001)^2] - [10 - 4.9(1)^2]}{0.0001}$$

$$\doteq -9.8\ m/s$$

$\therefore$ The rate of change is $-9.8\ m/s$

Ex 2:

The population, P, of wolves in a certain area can be modelled by the function P(t)= 500 + 100 sint, where t represents the time, in years. Estimate the instantaneous rate of change when t=6.

Are you in RAD?

$a = 6$ $h = 0.0001$
$a+h = 6.0001$

$$IRoC = \frac{f(a+h) - f(a)}{h}$$

$$= \frac{P(6.0001) - P(6)}{0.0001}$$

$$= \frac{[500 + 100\sin(6.0001)] - [500 + 100\sin(6)]}{0.0001}$$

$$\doteq 96.0$$

$\therefore$ The rate of change is 96 wolves/yr

Ex 3:
Determine the equation of the tangent of $f(x)=\dfrac{2x}{5-x^2}$ when x=-5

① Find slope

$$IRoC = \frac{f(-4.9999)-f(-5)}{0.0001}$$

$$= \frac{0.500015-0.5}{0.0001}$$

$$= 0.15$$

↳ Slope of the tangent

$m=0.15$

$f(-4.9999)$

$$= \frac{2(-4.9999)}{5-(-4.9999)^2}$$

$$= 0.500015$$

$f(-5)$

$$= \frac{2(-5)}{5-(-5)^2}$$

$$= 0.5$$

② Use slope and a point to solve for b in y=mx+b form

Point $(-5, 0.5)$

$m = 0.15$

$y = mx+b$

$0.5 = 0.15(-5)+b$

$1.25 = b$

$\therefore\ y = 0.15x + 1.25$

Homework:
p.87 #5-8
p.304 #5c,9