

1. Average Rate of Change = slope of secant
 [1] Instantaneous Rate of Change = slope of tangent

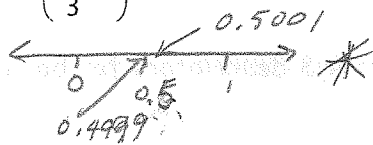
2. The population of a colony of rats is changing according to the model $P(t) = 5t^2 - 4t + 7$, where t is measured in minutes. Determine the average rate of change of population during the first 10 minutes. [3]

$$\begin{aligned} \text{ARoC} &= \frac{\Delta P}{\Delta t} \\ &= \frac{[5(10)^2 - 4(10) + 7] - [5(0)^2 - 4(0) + 7]}{10 - 0} \\ &= 46 \end{aligned}$$

\therefore average rate of change is 46 rats/min at 10 min

3. Estimate the instantaneous rate of change for the function

$f(t) = 50 - 20 \cos\left(\frac{8\pi}{3}t\right)$ when $t=0.5$ seconds using the preceding and following method. [3]



$$\begin{aligned} \Delta f &= f(0.5) - f(0.4999) \\ \frac{\Delta f}{\Delta t} &= \frac{[50 - 20 \cos\left(\frac{8\pi}{3}(0.5)\right)] - [50 - 20 \cos\left(\frac{8\pi}{3}(0.4999)\right)]}{0.5 - 0.4999} \\ &= -145.07 \end{aligned}$$

$$\begin{aligned} \Delta f &= f(0.5001) - f(0.5) \\ \frac{\Delta f}{\Delta t} &= \frac{[50 - 20 \cos\left(\frac{8\pi}{3}(0.5001)\right)] - [50 - 20 \cos\left(\frac{8\pi}{3}(0.5)\right)]}{0.5001 - 0.5} \\ &= -145.14 \end{aligned}$$

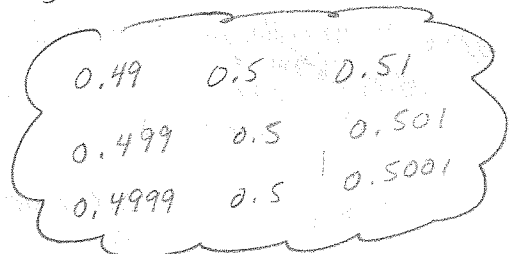
$$\begin{aligned} \text{IRoC} &= \frac{-145.14 + (-145.07)}{2} \\ &= -145.1 \end{aligned}$$

The IRoC is -145.01

4. Estimate the instantaneous rate of change for the function $f(x) = \frac{3x - 1}{x^2}$

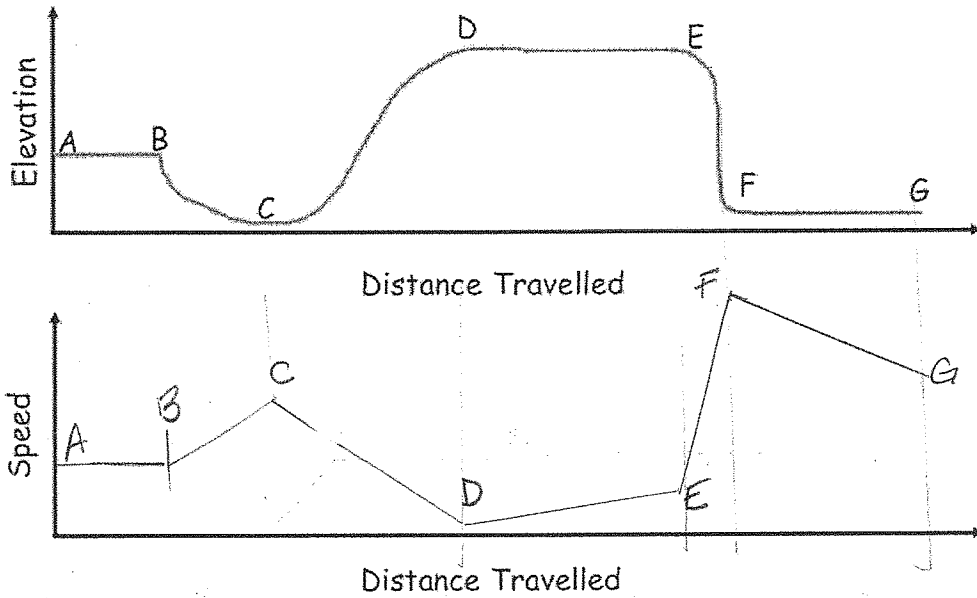
when $x=4$. Use the difference quotient method. [3] $a=4, h=0.0001$ *

$$\begin{aligned} \text{IRoC} &= \frac{f(a+h) - f(a)}{h} \\ &= \frac{\left[\frac{3(4.0001) - 1}{(4.0001)^2} \right] - \left[\frac{3(4) - 1}{(4)^2} \right]}{0.0001} \\ &= -0.156 \end{aligned}$$

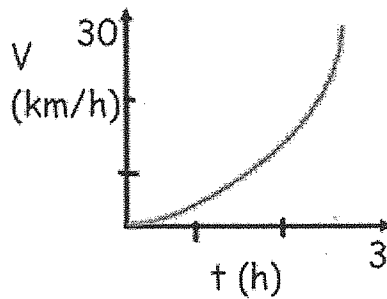
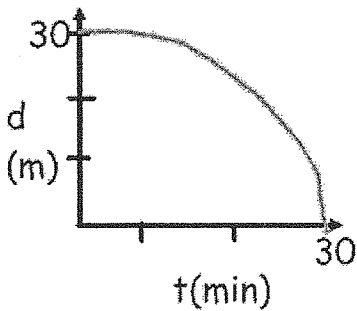


\therefore The instantaneous rate of change is -0.156

5. The picture shows the elevation of a cross-country ski trail. Sketch a graph of the possible speed of a skier versus the distance traveled along the trail. [3]



6. Describe the following graphs. Use at least 3 descriptors for both. [3]



negative tangent slopes = negative velocity

Slopes of tangents = negative = C.D.
 getting smaller (more negative) acceleration

$a \cdot v > 0$ = speeding up = slopes of tangents getting steeper
 positive displacement = above x-axis
 moving towards the origin

positive tangent slopes = positive acceleration

Slopes of tangents = acceleration increasing

positive velocity = above x-axis

$a \cdot v > 0$ = speeding up

Slopes of tangents