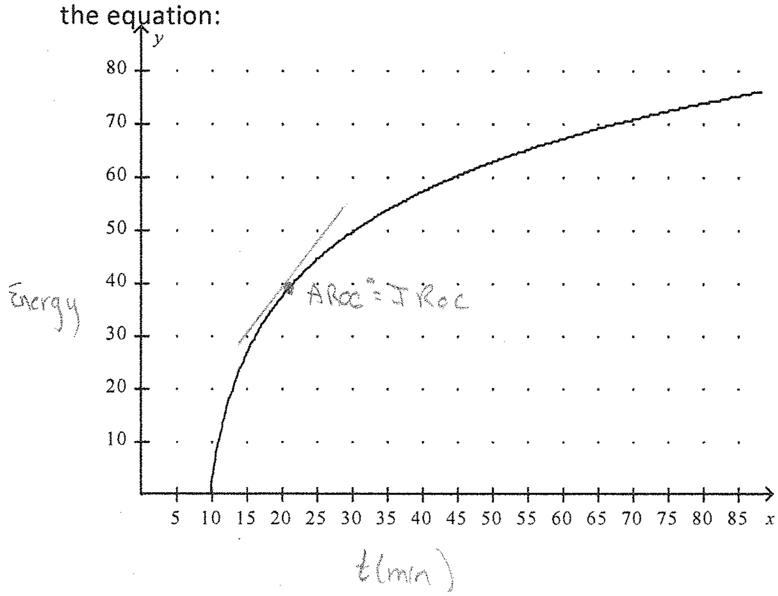
6.6 Rate of Change for Functions

Name: Sol^n.

A bird spends time foraging for food, then, having found a food source, perhaps a crab-apple tree, it begins to eat. At the beginning, the crab-apples are easy to find and the bird gains food energy at a high rate. As time passes, the tree is depleted, and the rate of energy gain decreases. At some point, the bird should leave the tree and search for a new one. Where E (calories) represents the energy obtained in the t minutes since foraging began, this relationship is modelled by the equation:



$$E(t) = \begin{cases} 0; t \leq 10 \\ \log_{105}(0.5t - 4); t \geq 10 \end{cases} \Rightarrow \frac{\log(0.5t-4)}{\log(1.05)}$$

1. Label the axes of the graph.
2. In this model, how long did it take the bird to find the crabapple tree?

   10 minutes to find the crabapples.

3. Calculate the average rate of change from:

a.	$t = 0$ to $t = 10$	0 cal/min	b.	$t = 0$ to $t = 15$	1.71 cal/min
c.	$t = 0$ to $t = 20$	1.84 cal/min	d.	$t = 0$ to $t = 30$	1.64 cal/min
e.	$t = 15$ to $t = 25$	1.82 cal/min	f.	$t = 60$ to $t = 70$	0.36 cal/min

4. Determine when the magnitude of the instantaneous rate of change is the highest. What does this mean?

   - when the slope of the tangent is steepest
   - when the bird first starts eating (decreases from there)

5. Determine the instantaneous rate of change at 6 and again at 10.

   IRoC(6) = 0 (no change) = 0 cal/min

   $IRoC(10) = \frac{E(10.0001) - E(10)}{0.0001} \approx 10.25$ cal/min

6. Determine the following:

   a. When is the instantaneous rate of change of the function 0? Before the bird finds food. $(0 < t < 10)$

   b. State the range of possible values of the instantaneous rate of change.

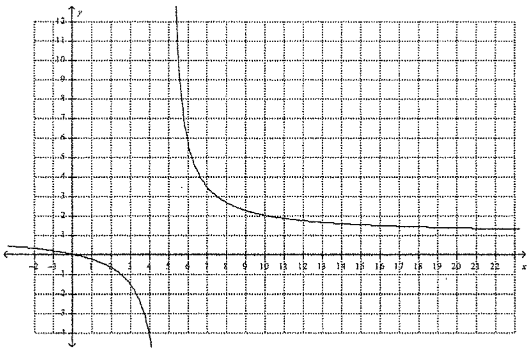
   $\{m_{tan} \in \mathbb{R} \mid 0 < m_{tan} < \infty\}$

7. According to this model, for how long should the bird stay at this tree before leaving to seek another?

   After 20 min when the instantaneous rate of change is less than the average rate of change.

MHF4U **6.6 Rate of Change for Functions** Name: Sol^n

The ratio of your age (in years) to your younger sibling's age (also in years) as time passes is modeled by the equation $r(x) = \frac{x}{x-5}$. A graphical representation is shown below.



Interpret each of these average rates of change in the context of the situation.

State the domain of the function for this situation. $D: \{x \in \mathbb{R} \mid x > 5\}$

1. What is the difference in your ages?

5 years. ∴ Function only exists for $x > 5$.

2. Calculate the average rate of change from

a) $x = 6$ to $x = 10$

$ARoC = \frac{r(10) - r(6)}{10 - 6} = -1$

a) $x = 0$ to $x = 6$

meaningless

$x = 80$ to $x = 90$

$ARoC = \frac{r(90) - r(80)}{90 - 80} = \frac{-1}{1275} \doteq -0.000784$

∴ the older we are the less the ratio changes

3. Determine when the magnitude of the instantaneous rate of change is 1. What does this mean?

$m_{tan} \neq 1$

never

$m_{tan} = -1$ @ $x = 7$

∴ when I am 7 yrs old the ratio changes by 1 yr.

4. Determine the instantaneous rate of change at 6 and again at 10.

$IRoC(6) = \frac{r(6.00001) - r(6)}{0.00001} \doteq -5$

$IRoC(10) = \frac{r(10.00001) - r(10)}{0.00001} \doteq -0.2$

5. Determine the following

a. When is the instantaneous rate of change of the function 0?

- never
- the slope of the tangent continues to approach zero but never reaches 0

b. When is the function's average rate of change 0?

- never
- it is not possible to draw a horizontal secant
- as $x \to \infty$ the secant slope gets closer to 0

b. Using set notation, state all possible values of the rate of change.

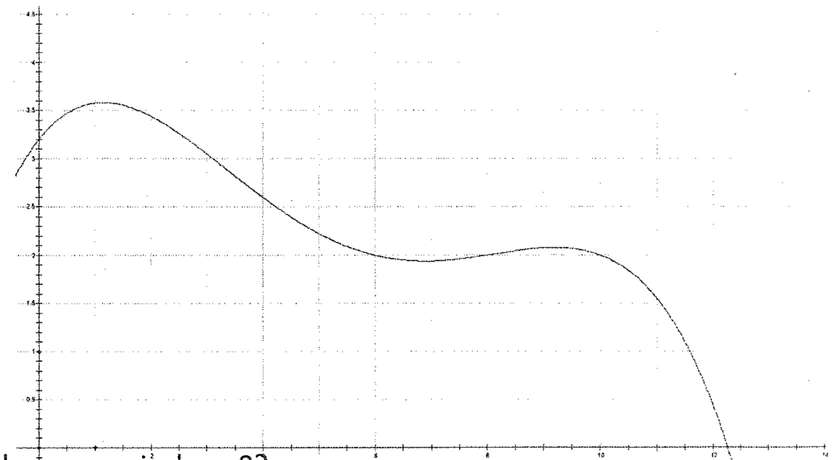
$RoC = \{m_{tan} \in \mathbb{R} \mid -\infty < m_{tan} < 0\}$

6 As the older sibling, what does the rate of change really mean? The answer may not be obvious...Think it through.

We get closer together as we age!
(but really ... I'm always 5 yrs older)

6.6 Rate of Change for Functions Name: Sol^n

The polynomial graph below models the height in feet above ground of a paper airplane versus time in seconds.



1. Use the graph to answer the following:
   a. When was the instantaneous velocity of the paper airplane 0?
   @ local max and local minimums

   b. Give at least three time intervals during which the paper airplane's average velocity was 0. Explain how you obtained these intervals.
   $0 \le t \le 2.5$
   $6 \le t \le 8$
   $8 \le t \le 10$
   these endpoints were obtained using horizontal secants (secants with end points that have equal y value).

   c. What was the average velocity of the paper airplane between 4s and 12 s?
   pts (4, 2.6) (12, 0.4)
   $ARoC = \frac{h(12)-h(4)}{12-4} = -0.44$
   ∴ average rate of change is -0.44 ft/sec.

   d. What was the average velocity of the paper airplane during its entire trip?
   pts (0, 3.2) (12.25, 0)
   $ARoC = \frac{h(12.25)-h(0)}{12.25-0} = \frac{0-3.2}{12.25-0} = -0.26$
   ∴ ARoC is -0.26 ft/sec.

   e. Estimate the times at which the instantaneous velocity of the paper airplane equals the average velocity of the paper airplane during its entire trip. Explain how you obtained these answers.
   drew secant from beginning to end of function, looked for tangent with the same slope.

   f. Estimate the initial velocity of the paper airplane.
   pt (0, 3.2) (2, 4.5)
   $m_{tan} = \frac{4.5-3.2}{2-0} = 0.65$
   ∴ initial velocity is 0.65 ft/sec

   g. Estimate the velocity of the airplane when it hits the ground.
   pt (12.25, 0) (11, 2.5)
   $m_{tan} = \frac{0-2.5}{12.25-11} = -2$
   ∴ hit ground @ -2 ft/sec.

   h. When was the instantaneous velocity of the paper airplane a maximum? Estimate this instantaneous velocity. where slope of tangent is steepest (@ points of inflection or end points).
   ∴ when it hit ground

   i. Use your answers from part a and parts e-h to sketch a graph of the velocity of the airplane versus time.