7.5a Modelling with Functions Jan 10th

All of the functions that you've studied in this course can be used to model real-life data....a function model will describe the data (not perfectly) and help to make predictions.

remember: linear, quadratic, cubic, quartic, exponential, logarithmic, rational, root, absolute value, trigonometric, inverses........and combinations!!!!!

The Logistic Function NEW!!

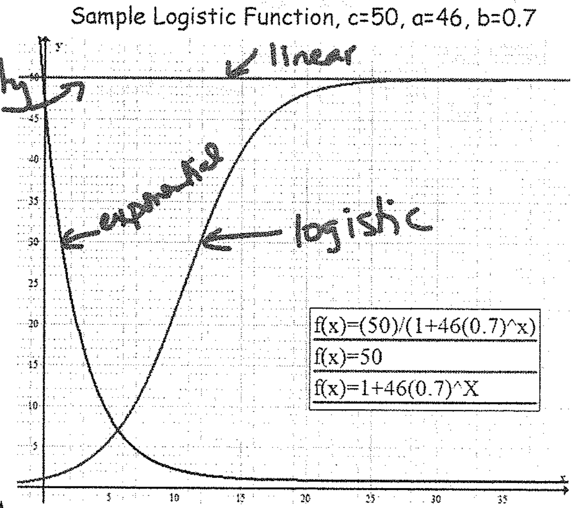
$$P(t)= \frac{c}{1+ab^t}$$

c ← linear function, c=maximum value

$1+ab^t$ ← exponential function, t= time

Sample Logistic Function, c=50, a=46, b=0.7

carrying capacity



- a common and widely used model for growth
- it models slow growth for small values of t
- then rapid growth
- then slow growth again until it reaches a maximum value (max. at horiz. asympt)
- the values of a and b can be found using any 2 points on the function

☆

PART A: Modelling by Hand

Ex: #1

A pond study in the back 40 has revealed that the population of a water bug that was initially 30 has grown to 240 in 5 days. If the maximum capacity of the pond is 1000 of these bugs, how long will it take to reach the maximum? Let x represent time in days & let y represent # of bugs.

a) use a linear model (y=mx+b)

(0,30) (5,240)

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $b = 30$

$= \frac{240-30}{5-0}$

$= 42$

$\therefore y = 42x + 30$

Subst $y=1000$ & solve for x

$1000 = 42x + 30$

$\frac{1000-30}{42} = x$ $\therefore x \doteq 23$ days

b) use an exponential model ($y=ca^x$)

initial is 30 $\therefore c = 30$

(5,240)
x y

$y = c \cdot a^x$

$(240) = (30)\, a^{(5)}$

$\frac{240}{30} = a^5$

$8 = a^5$

$a = \sqrt[5]{8}$

$\doteq 1.52$ Need to keep in calc.

$\therefore y = 30 \cdot (1.52)^x$

$1000 = 30 \cdot (1.52)^x$

$\frac{1000}{30} = 1.52^x$

$\frac{100}{3} = 1.52^x$

$\log\left(\frac{100}{3}\right) = x \log(1.52)$

$x = \frac{\log\left(\frac{100}{3}\right)}{\log(1.52)}$

$\doteq 8.4$ days.

c) use a logistic model

$P(t)=\frac{c}{1+ab^t}$ $y=\frac{c}{1+ab^x}$

$(0,30)$

$(5,240)$

$x \quad y$

$c=1000$

$30=\frac{c}{1+ab^0}$

$30(1+a)=c$

$30+30a=c$

$30+30a=1000$

$30a=970$

$a \doteq 32.3 \leftarrow$ keep in calc.

$y=\frac{c}{1+ab^x}$

$240=\frac{1000}{1+(32.3)(b)^5}$

$1000=240(1+(32.3)b^5)$

$1000=240+7759.99b^5$

$\frac{760}{7759.99}=\frac{7759.99b^5}{7759.99}$

$b^5=0.098$

$b \doteq 0.63$ keep

$y=\frac{c}{1+ab^x}$

$=\frac{1000}{1+(32.3)(0.63)^x}$

HA pick 999.99

subst $y=1000$

$999.99=\frac{1000}{1+(32.3)(0.63)^x}$

$1+(32.3)(0.63)^x=\frac{1000}{999.99}$

$(32.3)(0.63)^x=\frac{1000}{999.99}-1$

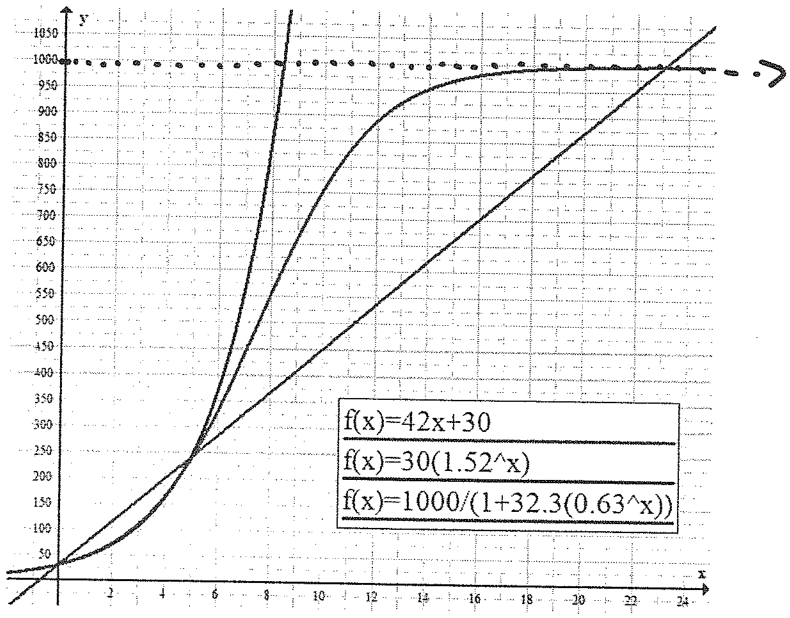
$(0.63)^x=\left(\frac{1000}{999.99}-1\right)\div 32.3$

$x\cdot\log(0.63)=\log\left(\frac{\frac{1000}{999.99}-1}{32.3}\right)$

$x \doteq 32.4$ days

(37.5 days)

Graphs from Ex. #1:



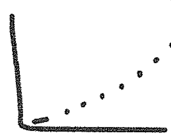
Discuss: Which is a better model????

Part B: Modelling Using Graphing Technology

Ex: 2 The table below shows the amount of water vapour (mL of water/m^3 of air) in the air as a function of temperature (°C).

Temp	Vapour
0	4.8
5	6.8
10	9.4
15	12.8
20	17.3
25	23.1
30	30.4
35	39.6

a) Create a scatter plot of the data.



b) Use regression to find the equation of each type of model and note the value of R^2:

↳ coefficient corelation

linear: $y \doteq 0.97x + 1.06$

$R \doteq 0.94$

quadratic: $y = 0.02x^2 + 0.14x + 5.23$

$R^2 \doteq 0.999$

unless domain is restricted

cubic: $y = 0.0003x^3 + 0.0066x^2 + 0.36x + 4.8$

$r^2 \doteq 0.9999....$

does not make sense for the question

exponential: $y = 5.05(1.06)^x$

$r^2 \doteq 0.999$

other???

R^2: this value is the fraction of variance in y that is explained by the model based on x. In general, values closer to 1.0 (ie. 100%) are better fits......though this is NOT always the case. The model must also MAKE SENSE with the given data.

Homework
page 569
#1,2,4,5,12... pencil & paper

#7,8,11, **need graphing technology