
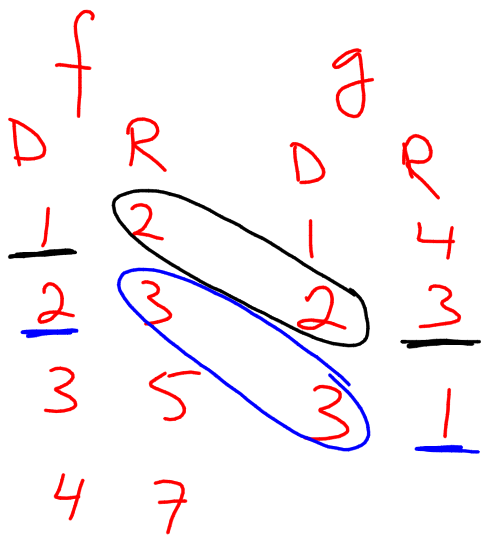


Example 1: Let $f = \{(1,2), (2,3), (3,5), (4,7)\}$ and $g = \{(1,4), (2,3), (3,1)\}$, determine the following:

| | | |
|---------------------|---------------------|---------------------|
| a) $(f \circ g)(2)$ | b) $(g \circ f)(2)$ | c) $(f \circ f)(1)$ |
| $= f(g(2))$ | $= g(f(2))$ | $= f(f(1))$ |
| $= f(3)$ | $= g(3)$ | $= f(2)$ |
| $= 5$ | $= 1$ | $= 3$ |

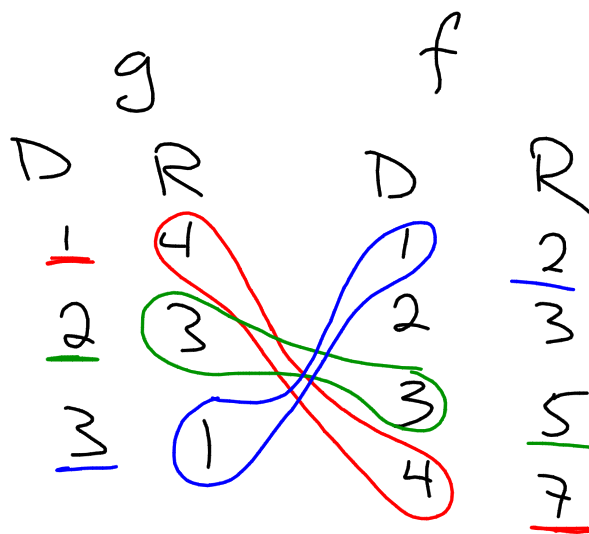
 NOTE: composition is not commutative
 $f \circ g \neq g \circ f$

d) $g \circ f$



$$g \circ f = \{(1, 3), (2, 1)\}$$

e) $f \circ g = f(g(x))$



$$f \circ g = \{(1, 7), (2, 5), (3, 2)\}$$

Example 2: Let $f(x) = 4x+1$, $g(x) = \sqrt{x+2}$, $m(x) = \cos x$, $h(x) = 3^x - 5$
 Determine the domain and range and...

| | | |
|---------------------|---------------------|------------------------|
| a) $(g \circ f)(x)$ | b) $(h \circ m)(x)$ | c) $(g \circ h)(x)$ |
| $= g(f(x))$ | $= h(m(x))$ | $= g(h(x))$ |
| $= g(4x+1)$ | $= h(\cos x)$ | $= g(3^x - 5)$ |
| $= \sqrt{(4x+1)+2}$ | $= 3^{\cos x} - 5$ | $= \sqrt{(3^x - 5)+2}$ |
| $= \sqrt{4x+3}$ | | $= \sqrt{3^x - 3}$ |

$$4x+3 \geq 0$$

$$4x \geq -3$$

$$x \geq -\frac{3}{4}$$

$$D = \{x \in \mathbb{R} \mid x \geq -\frac{3}{4}\}$$

$$R = \{y \in \mathbb{R} \mid y \geq 0\}$$

$$D = \{x \in \mathbb{R}\}$$

$$R = \{y \in \mathbb{R} \mid -\frac{14}{3} \leq y \leq -2\}$$

Possible values of $\cos x$?

| | |
|--------------------|-------------------|
| $-\frac{1}{3}$ | $\frac{1}{3}$ |
| $-\frac{1}{3} - 5$ | $\frac{1}{3} - 5$ |
| $= -\frac{14}{3}$ | $= -2$ |

$$3^x - 3 \geq 0$$

$$3^x \geq 3$$

$$x \geq 1$$

$$D = \{x \in \mathbb{R} \mid x \geq 1\}$$

$$R = \{y \in \mathbb{R} \mid y \geq 0\}$$

THINKIN'

What happens when a function is composed with its inverse?

Try: $(f \circ f^{-1})(x)$ $f(x) = 4x+1$

$$x = 4y+1 \Rightarrow f^{-1}(x) = \frac{x-1}{4}$$

$$y = \frac{x-1}{4}$$



$$(f \circ f^{-1})(x) = f(f^{-1}(x))$$

$$= f\left(\frac{x-1}{4}\right)$$

$$= 4\left(\frac{x-1}{4}\right) + 1$$

$$= x - 1 + 1$$

$$= x$$

★ Recall that the inverse function "undoes" the effect of the original function. As a result, the domain of the original function becomes the range of the inverse. The range of the original function becomes the domain of the inverse.

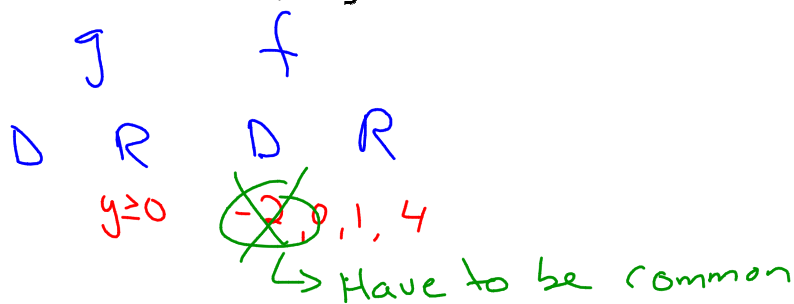
You put in "x" you get out "x".

Example 3: For each function, find two functions f and g such that $h(x) = f(g(x))$.

- a) $h(x) = \sqrt{5x-3}$ b) $h(x) = 2^{x^2-6x}$
 Inner function: $g(x) = 5x-3$ Inner: $g(x) = x^2-6x$
 Outer function: $f(x) = \sqrt{x}$ Outer: $f(x) = 2^x$

Example 4: Let $f = \{(-2,1), (0,4), (1,2), (4,5)\}$ and $g(x) = \sqrt{x-3}$
 Evaluate:

- a) $(f \circ g)(3) = f(g(3)) = f(\sqrt{3-3}) = f(0) = 4$
 b) $(f \circ g)(7) = f(g(7)) = f(\sqrt{7-3}) = f(2) = \text{UNDEFINED!}$
 c) determine the domain of $f \circ g$

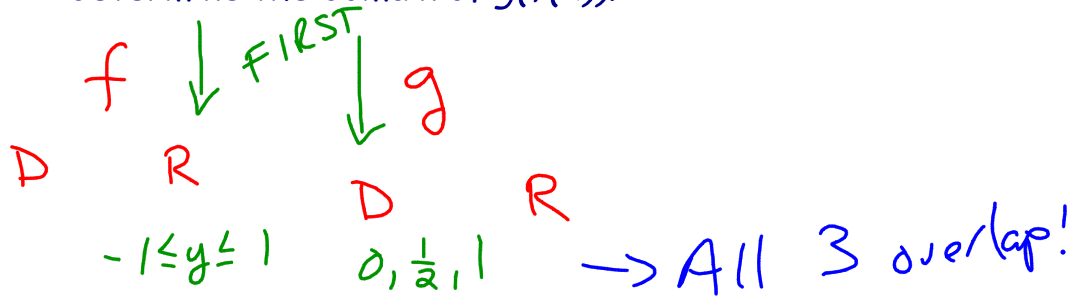


Find domain of g by using Domain (x-values) of f as Range (y-values) for g

| | | |
|------------------|------------------|------------------|
| <u>0</u> | <u>1</u> | <u>4</u> |
| $0 = \sqrt{x-3}$ | $1 = \sqrt{x-3}$ | $4 = \sqrt{x-3}$ |
| $0 = x-3$ | $1 = x-3$ | $16 = x-3$ |
| $3 = x$ | $4 = x$ | $19 = x$ |

$D_{f \circ g} = \{3, 4, 19\}$

Example 5: Given $f(x) = \sin x$ and $g(x) = \{(0, -5), (\frac{1}{2}, 0), (1, 5)\}$, determine the domain of $g(f(x))$.



Values of f that give $0, \frac{1}{2}, 1$?

$f = \sin(x)$

$$\frac{0}{0 = \sin x}$$

$$x = \pi n, n \in \mathbb{Z}$$

$$\frac{\frac{1}{2}}{\frac{1}{2} = \sin x}$$

$$x_R = \frac{\pi}{6}$$

$$\begin{array}{c} \checkmark \\ \checkmark \end{array}$$

$$\frac{Q1}{x = \frac{\pi}{6} + 2\pi n}$$

$$\frac{Q2}{x = \frac{5\pi}{6} + 2\pi n}$$

$$\frac{1}{1 = \sin x}$$

$$x = \frac{\pi}{2} + 2\pi n$$

$$D_{g \circ f} = \left\{ \pi n, \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n, \frac{\pi}{2} + 2\pi n \right\},$$

$$n \in \mathbb{Z}$$

Homework:

Pg 552 #1bcde, 2ace, 3, 6(all), 7df, 8, 12, 14

mistakes

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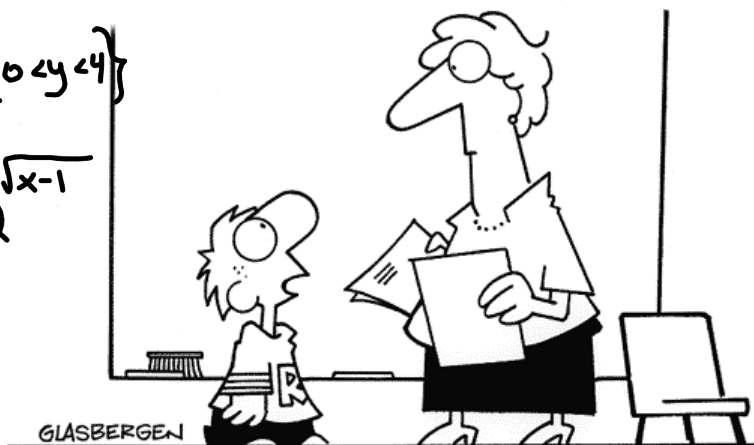
$$6c) R_{g \circ f} = \{0 < y < 4\}$$

$$6d) f \circ g = 2^{\sqrt{x-1}}$$

*

$$\sqrt{16-x^4}$$

$$D = \{-2 \leq x \leq 2\}$$



GLASBERGEN

"I couldn't do my homework because my computer has a virus and so do all my pencils and pens."