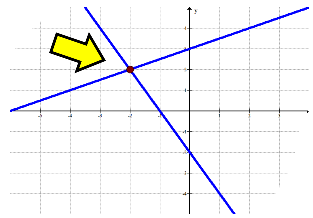
1.4 Solving Systems by Substitution

Recall: Solving a system of equations means...

Finding the point of intersection of the lines.

It is the only point where both lines have the same x-value and y-value.



Drawbacks of solving by graphing...

- not always accurate
- time consuming

What's so great about solving algebraically?

- gives exact values
- less time / less space

Ex. 1 Rogers charges \$35/month plus \$10 for every extra Gb. Bell charges \$40/month plus \$8 for every extra Gb. When are they the same price? Solve without graphing.

Let C be the cost in dollars
Let g be the # of GB

① $C_R = 10g + 35$
② $C_B = 8g + 40$

WANT TO FIND WHEN THEY ARE THE SAME! GRAPH

$C_R = C_B$

$10g + 35 = 8g + 40$
$10g - 8g = 40 - 35$
$2g = 5$
$g = \frac{5}{2}$

Sub $g = \frac{5}{2}$ into ①
$C = \cancel{10}^{5}\left(\frac{5}{\cancel{2}}\right) + 35$
$= 25 + 35$
$= 60$

∴ The plans are equal @ \$60 when using 2.5GB

Ex. 2 How would you solve
① x = 5
② 3x - 4y = 3

Sub ① into ②

$3(5) - 4y = 3$
$15 - 4y = 3$
$-4y = -12$
$y = 3$

∴ The Sol^{n} is (5, 3)

THE SUBSTITUTION METHOD:

1. **Isolate** a variable in one equation (pick the ~~best~~ easiest one)

2. **Substitute** to create an equation with only one variable.

3. **Solve** the equation.

4. ***Substitute*** the solved variable into the equation from #1 to determine the value of the other variable.

5. **Write** a conclusion.

6. (**Check**) - formal if asked, otherwise complete a mental check.

Ex. 3 Solve using the substitution method.

a) $x + 3y = -4$ ①
$2x - 3y = 1$ ②

① $x = -4 - 3y$

Sub into ②

$2(-4-3y) - 3y = 1$

$-8 - 6y - 3y = 1$

$-8 - 9y = 1$

$-9y = 9$

$y = -1$

Sub $y = -1$ into ①

$x + 3(-1) = -4$

$x - 3 = -4$

$x = -1$

$\therefore$ Soln is $(-1, -1)$

b) $5a + 3b = 10$ ①
$2a - b = 4$ ②

$2a - 4 = b$ $\quad b = 2a - 4$

Sub ② into ①

$5a + 3(2a-4) = 10$

$5a + 6a - 12 = 10$

$11a = 22$

$a = 2$

Sub $a = 2$ back into ②

$2(2) - b = 4$

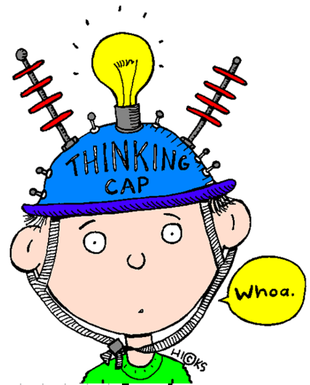
$4 - b = 4$

$-b = 0$

$b = 0$

$\therefore$ Soln is $a = 2$, $b = 0$

SPECIAL CASES



A.

$x + y = 8$ ①
$3x + 3y = -5$ ②

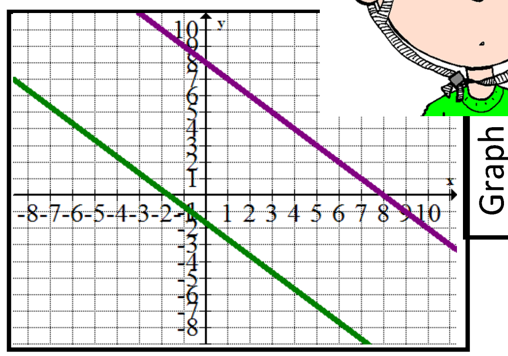
① $x = 8 - y$

Sub into ②

$3(8-y) + 3y = -5$

$24 - 3y + 3y = -5$

$24 = -5$



∴ NOT POSSIBLE

B. $p - 2q = -3$ ①
$4q = 2p + 6$ ②

① $p = -3 + 2q$

Sub into ②

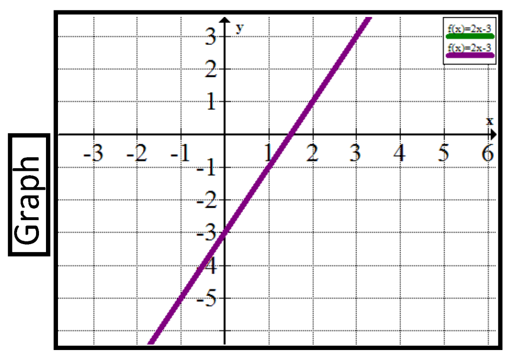
$4q = 2(-3+2q) + 6$

$4q = -6 + 4q + 6$

$4q = 4q$

(OR)

$0 = 0$



∴ Always TRUE → MANY SOLUTIONS
(SAME LINE)

Working with fractions...
pg. 26 #4a)

① $x + 2y = 3$
② $5x + 4y = 8$

① $x = 3 - 2y$

Sub into ②

$5(3-2y) + 4y = 8$

$15 - 10y + 4y = 8$

$-6y = -7$

$y = \frac{7}{6}$

Sub $y = \frac{7}{6}$ into ①

$x + \cancel{2}^{1}\left(\frac{7}{\cancel{6}_{3}}\right) = 3$

$x + \frac{7}{3} = 3$

$x = 3 - \frac{7}{3}$

$= \frac{9}{3} - \frac{7}{3}$

$= \frac{2}{3}$

$\therefore$ The $\text{sol}^{\underline{n}}$ is $\left(\frac{2}{3}, \frac{7}{6}\right)$

The following three lines all intersect at one point. Find the coordinates of the point of intersection and the value of *k*.

$$2x + 3y = 7$$
$$x + 4y = 16$$
$$4x - ky = 9$$

Your Turn

p. 26 #4bd, 5ace, 12, 19