4.3 The Sine Law in Acute Triangles

The Sine Law is True in any acute triangle:

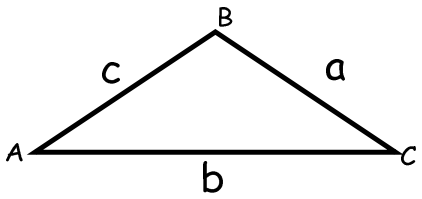
for finding sides:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

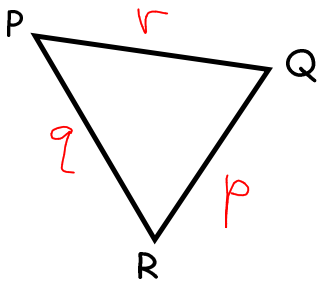
for finding angles:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Pay attention at the labeling of a triangle:



The vertices are labeled in capitals, and each side corresponds to the opposite vertex, labeled in lower case letter.



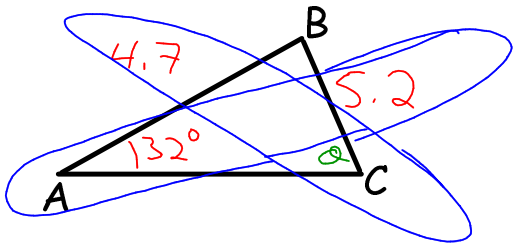
Ex 1:

Given $\triangle ABC$ where: $c=4.7$ Find: $\angle C$

$a=5.2$

$\angle A=32°$

Solution:



$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin\theta}{4.7} = \frac{\sin 32°}{5.2}$$

$$\sin\theta = 4.7 \cdot \frac{\sin 32°}{5.2}$$

$$\sin\theta = 0.4790$$

$$\theta = \sin^{-1}(0.4790)$$

$$\therefore 29°$$

$\angle C = 29°$

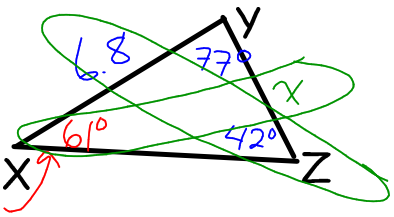
Ex 2:

In a triangle XYZ the following is given:

$z=6.8$ cm, $\angle Z=42°$, $\angle Y=77°$

Solve triangle XYZ.

(To solve a triangle means to find ALL unknown sides and angles)



$180-77-42$

$\frac{x}{\sin X}=\frac{8}{\sin Z}$

$\frac{x}{\sin 61°}=\frac{6.8}{\sin 42°}$

$x=\sin 61° \cdot \frac{6.8}{\sin 42°}$

$\boxed{x \doteq 8.89}$

$\frac{y}{\sin Y}=\frac{8}{\sin Z}$

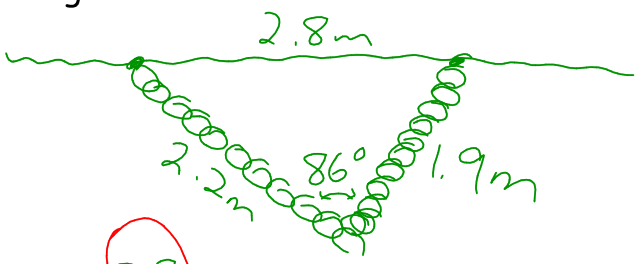
$\frac{y}{\sin 77°}=\frac{6.8}{\sin 42°}$

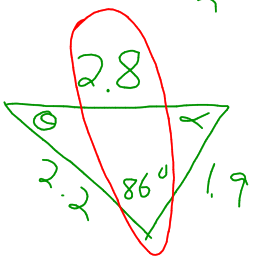
$y=\sin 77° \cdot \frac{6.8}{\sin 42°}$

$\boxed{y \doteq 9.90}$

Ex 3:

Toby uses chains and a winch to lift engines at his garage. Two hooks in the ceiling are 2.8 m apart. each hook has a chain hanging from it. The chains are of 1.9 m and 2.2 m. When the ends of the chains are attached, they form an angle of 86° . In this configuration, what acute angle to the nearest degree does each chain make with the ceiling?





$\frac{\sin\theta}{1.9} = \frac{\sin 86^\circ}{2.8}$

$\sin\theta = 1.9 \cdot \frac{\sin 86^\circ}{2.8}$

$\sin\theta \doteq 0.6769$

$\theta \doteq 43^\circ$

$\alpha = 180^\circ - 86^\circ - 43^\circ$

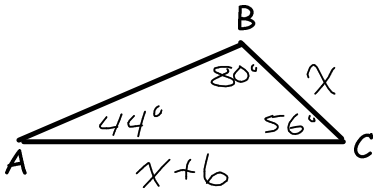
$= 51^\circ$

∴ the 1.9m chain makes an angle of 51°

the 2.2m " " " " " 43°

Ex 4:

Two angles in a triangle measure 56° and 44°. The longest side of the triangle is 6 cm longer than the shortest side. Calculate the shortest and longest sides.



Hint: The longest side is across from the largest angle
and
The shortest side is across from the smallest angle

$\theta = 180 - 56° - 44°$
$= 80°$

Need to find x!

$$\frac{x}{\sin 44°} = \frac{x+6}{\sin 80°}$$

$$\frac{a}{b} \times \frac{c}{d}$$

$ad = cb$

$$x = \sin 44° \cdot \frac{x+6}{\sin 80°}$$

$\sin 80° \cdot x = \sin 44° \cdot (x+6)$

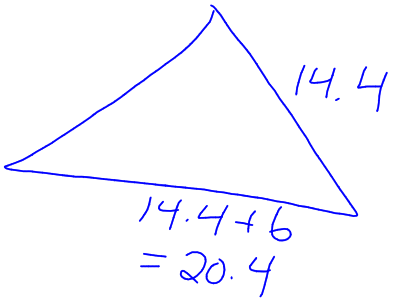
$0.9848x = 0.6947(x+6)$

$0.9848x = 0.6947x + 4.1682$

$0.2901x = 4.1682$

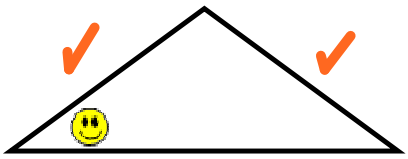
$$x = \frac{4.1682}{0.2901}$$

$= 14.4$

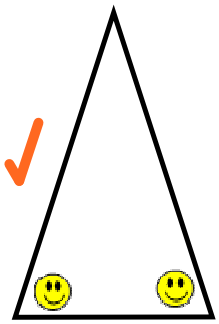


∴ The shortest side is 14.4cm
The longest side is 20.4cm

Use the Sine Law when you have an oblique (non right) triangle that has:



- two sides and the angle across from one known side



- two angles and <u>any</u> side

(note: you can find the third angle by subtracting from 180° if you need it)


Hmwk p 288 # 3, 5, 6a, 7, 8c, 9c, 10,13