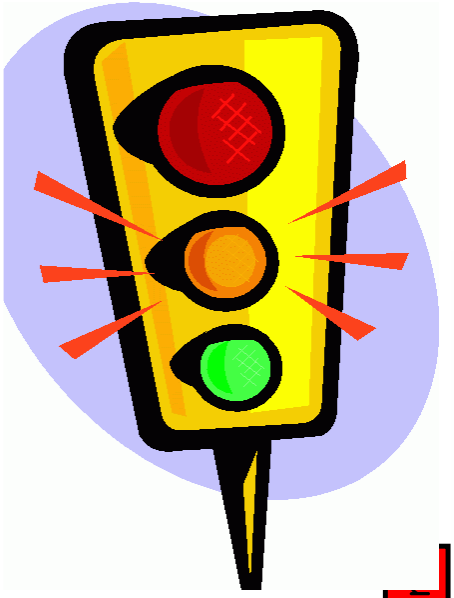
6.3 Rational Exponents $x^{\frac{1}{n}}$



Ready: 10 minute warm up

Set: Let's investigate

Go: Rational Exponent note

Warm-up Answer

1. a) x^7 c) m^8 e) y^6
   b) p^2 d) $\frac{1}{a^2}$ f) $\frac{1}{k^{12}}$
2. a) $\frac{1}{y^2}$ c) $\frac{1}{n^{24}}$ e) 1
   b) x^4 d) $\frac{1}{w}$ f) $\frac{1}{b^{19}}$
3. a) 36 b) $x^2y^2 - 36$
   c) Usually it is faster to substitute numbers into the simplified form.
4. a) p^2q c) $\frac{1}{a^2b^7}$ e) wx^4
   b) $\frac{y^2}{x^6}$ d) $\frac{n^6}{m^4}$ f) $\frac{a^6}{b^4}$
5. a) $72x^8y^{11}$ c) $\frac{y^6}{150x^4}$ e) $\frac{r^4}{p^7}$
   b) $\frac{a^5}{b^2}$ d) $\frac{3m^{10}}{4n^2}$ f) $\frac{y^4}{x^5}$

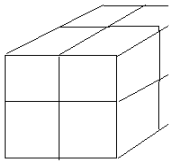
6.3 Investigating Rational Exponents $x^{\frac{1}{n}}$

1. a) Use the fact that $\sqrt{9} = 3$ to complete the following.

$\sqrt{9} \times \sqrt{9} = \boxed{3} \times \boxed{3} = \boxed{9}$

b) Use the law of exponents for multiplication to complete the following

$9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\boxed{\frac{1}{2}} + \boxed{\frac{1}{2}}} = 9^{\boxed{1}}$



c) compare the statements in a) and b). What other mathematical operation does the exponent $\frac{1}{2}$ seem to be equivalent to?

2. Because $2^3 = 8$ (two cubed equals eight), we say that the cube root of 8 is 2,

and we write $\sqrt[3]{8} = 2$

a) Use the fact that $\sqrt[3]{8} = 2$ to complete the following.

$\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = \square \times \square \times \square = \square$

b) use the law of exponents for multiplication to complete the following.

$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{\square + \square + \square} = 8^{\square}$

c) compare the statements in a) and b). What other mathematical operation does the exponent $\frac{1}{3}$ seem to be equivalent to?

3. Extend your reasoning to make a generalization about the meaning of $x^{\frac{1}{n}}$ (where $x>0$ and n is a natural number)

6.3 Working With Rational Exponents

Exponent Law

radical sign $\sqrt{\ }$

Exponential form → radical form

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$ (index → n)

index: indicates what root you want

Evaluate:

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$9^{\frac{1}{2}} = \sqrt{9} = 3$

Note: when the index is 2 we don't write it... it is understood (square root)

Extend the rule:

numerator is the exponent

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

denominator is the index

OR

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Ex 1: Evaluate

$$125^{\frac{2}{3}}$$

Does order matter?

method 1

$$\sqrt[3]{125^2}$$

$$= \sqrt[3]{15625}$$

$$= 25$$

method 2

$$\left(\sqrt[3]{125}\right)^2$$

$$= (5)^2$$

$$= 25$$

Quotient of bases with Rational Exponents

Ex 2: Evaluate

$$\frac{50^{\frac{1}{2}}}{2^{\frac{1}{2}}}$$

Does order matter?

method 1

$$\frac{\sqrt{50}}{\sqrt{2}}$$

$$= \frac{7.071}{1.414}$$

★ DO THIS ALL AT ONCE ON CALC

$$= 5$$

method 2

$$\left(\frac{50}{2}\right)^{\frac{1}{2}}$$

$$= 25^{\frac{1}{2}}$$

$$= 5$$

General Rule:

$$\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}} = \left(\frac{a}{b}\right)^{\frac{1}{2}} = \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Product of radicals

Ex 3: Evaluate

Does order matter?

$16^{\frac{1}{3}} \times 4^{\frac{1}{3}}$

method 1

$\sqrt[3]{16} \times \sqrt[3]{4}$

$= 4$

method 2

$(16 \times 4)^{\frac{1}{3}}$

$= 64^{\frac{1}{3}}$

$= 4$

Alternate Method: create like bases

$(4^2)^{\frac{1}{3}} \times 4^{\frac{1}{3}}$

$= 4^{\frac{2}{3}} \cdot 4^{\frac{1}{3}}$

$= 4^1$

General Rule:

$$a^{\frac{1}{n}} \times b^{\frac{1}{n}} \times c^{\frac{1}{n}} = \sqrt[n]{a} \times \sqrt[n]{b} \times \sqrt[n]{c} = \sqrt[n]{a \times b \times c}$$

Practice:

$121^{\frac{1}{2}} = \sqrt{121} = 11$

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$64^{\frac{2}{3}}$

$(64^2)^{\frac{1}{3}} = 4096^{\frac{1}{3}} = 16$

$(64^{\frac{1}{3}})^2 = 4^2 = 16$

$(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$

$32^{-\frac{1}{5}} = \frac{1}{32^{\frac{1}{5}}} = \frac{1}{2}$

$-81^{\frac{3}{4}} = -(81^{\frac{1}{4}})^3 = -(3)^3 = -27$

$\frac{\sqrt{12}}{\sqrt{8} \times \sqrt{6}} = \sqrt{\frac{12}{8 \cdot 6}} = \sqrt{\frac{12}{48}} = \sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$

Ex 4: Simplify then evaluate:

$$\frac{\left(8^{\frac{1}{6}}\right)^{7}}{8^{\frac{1}{2}}\ 8^{\frac{1}{3}}}$$

$= \frac{8^{\frac{7}{6}}}{8^{\frac{5}{6}}}$

$= 8^{\frac{7}{6}-\frac{5}{6}}$

$= 8^{\frac{2}{6}}$

$= 8^{\frac{1}{3}}$

$= 2$

$\frac{1}{2} + \frac{1}{3}$

$= \frac{3}{6} + \frac{2}{6}$

$= \frac{5}{6}$

Practice:

Simplify the following expressions:

a) $\dfrac{(m^5)^{-\frac{9}{5}}}{\left(m^{-\frac{3}{2}}\right)^4}$

$5\left(-\frac{9}{5}\right)$

$-\frac{3}{2}(4) = -6$

$= \frac{m^{-9}}{m^{-6}} \rightarrow = m^{-9-(-6)}$

$= m^{-3}$

$= \frac{1}{m^3}$

$= \frac{m^6}{m^9}$

$= \frac{1}{m^3}$

b) $\dfrac{m^{\frac{1}{2}} \cdot m^{\frac{3}{2}}}{\left(m^{-2}\right)^{\frac{1}{2}}}$

$= \frac{m^{\frac{1}{2}+\frac{3}{2}}}{m^{-\frac{2}{2}}}$

$= \frac{m^2}{m^{-1}}$

$= m^2 \cdot m^1$

$= m^3$

Practice:

p 415 # 1, 4, 6,

[9 - 11, 15,17](def),

20a