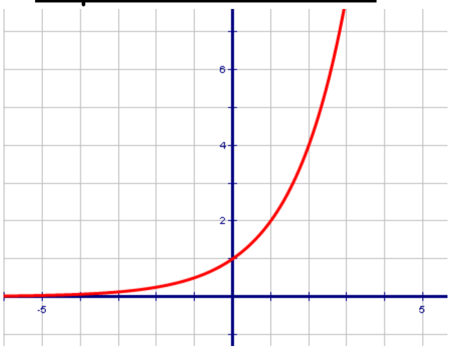
6.6 Intro to Exponential Growth and Decay

Summary:

function:	algebraic model:
Linear - constant 1st diff.	$y = mx + b$
Quadratic - constant 2nd diff.	$y = x^2$
Exponential - Constant Ratio of 1st diff.	$y = ca^x$

Exponential Growth:



Graph:
Increases slowly then quickly

Exponential Decay:

Graph:
Decreases quickly then slowly

Asymptote:
A line that a curve will continually approach but never touch

Asymptote:
- x axis
- eq'n: y=0

y will never be zero!

NOTE: for our purposes we will not be looking at a graph that has been translated up or down, that is why the x axis will be the asymptote

Equation:

$$y = ca^x$$

exponent is a variable

(stretch) (initial value)

base (growth/decay factor)

- y intercept is 1 if there is no stretch
  i.e. c = 1
- your base is the constant ratio of the 1st differences
- from your graph you can see your base by finding your y value at x = 1
  * watch for a stretch*
- Exponential functions need a positive base

The following table shows exponential growth:

x	f(x)	
0	1.0000	$\times 5$
1	5.0000	$\times 5$
2	25.0000	$\times 5$
3	125.0000	$\times 5$
4	625.0000	$\times 5$
5	3125.0000	

c

a

Growth is exponential if there is a common ratio for consecutive values in the table.

(we sometimes have to look at first differences to see the ratio)

(Recall linear relations have a common first difference and Quadratic relations have a common second difference)

$$y = c\,a^x$$
$$= 1 \cdot 5^x$$
$$= 5^x$$

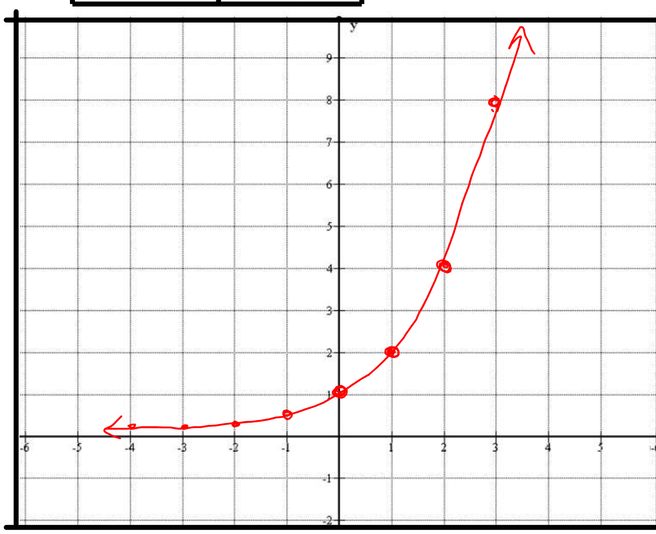
Complete the table of values and graph each

$y = 2^x$

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16

$y = \left(\frac{1}{2}\right)^x$

x	y
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$
4	$\frac{1}{16}$



$y = 2^x$

Exp. GROWTH

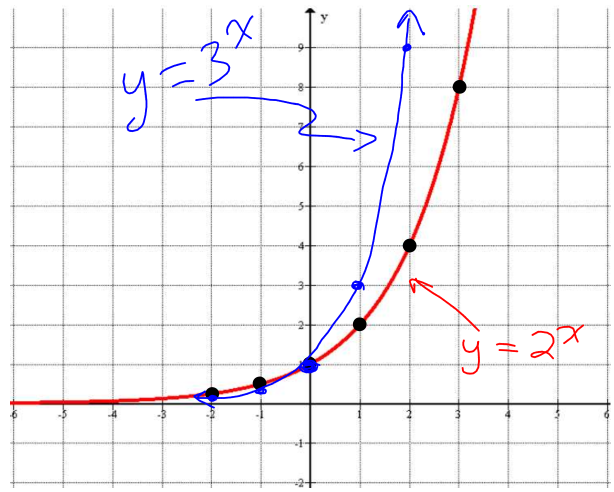
Base > 1

$y = \left(\frac{1}{2}\right)^x$

Exp. DECAY

$0 < \text{Base} < 1$

The graph of $y = 2^x$ is shown to the right



Complete the table and graph $y = 3^x$ on the same grid

x	y
-3	1/27
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27
4	81

☆ Look at the value of y when x=1
What do you notice?

What are the similarities between the graph of $y=2^x$ and $y=3^x$

- Both are exp. growth
- Both have a y int. of 1

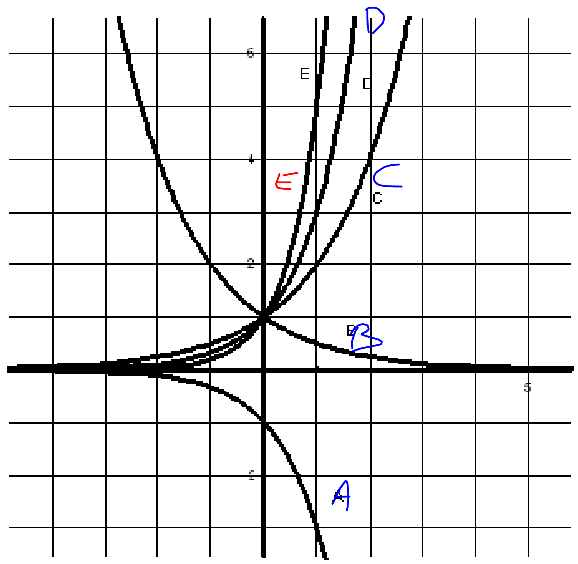
What are the differences between the graph of $y=2^x$ and $y=3^x$

$y=3^x$ increases more quickly

Match the Graph to the Exponential Equation

(Hint: To find the appropriate base find the value of y when $x = 1$)

1.



$y = 3^x$ D

$y = 2^x$ C

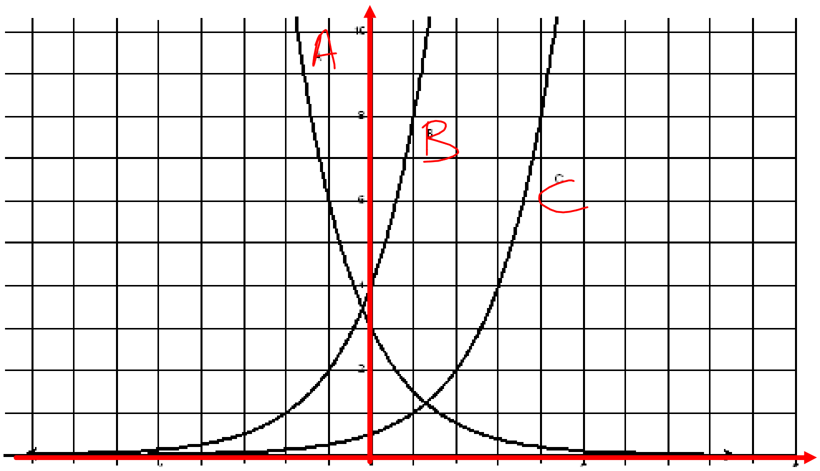
$y = \left(\frac{1}{2}\right)^x$ B

$y = \left(\frac{1}{3}\right)^x$ DNE

$y = 5^x$ E

$y = -3^x$ A

2.



Notice the initial value

$y = 4(2)^x$ B

$y = \frac{1}{2}(2)^x$ C

$y = 3\left(\frac{1}{2}\right)^x$ A

$y = (-6)^x$ ______

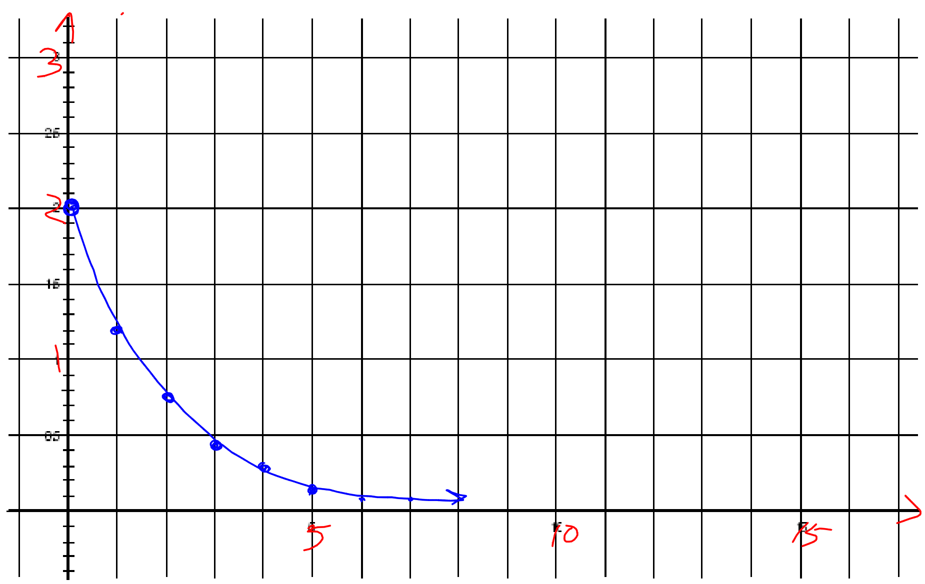
Real World Applications

3. The height of a ball in metres, h, after n bounces can be modeled by the equation $h = 2(0.6)^n$.

i) Create a table of values.
Start at 0 bounces and round your height to 2 decimals.

ii) Now graph the relation. (Be careful it is a **curve**, not a straight line- do NOT use a ruler)
Be sure to Title label the axis on the graph.

n	h
0	2
1	1.2
2	0.72
3	0.43
4	0.26
5	0.16
6	0.09
7	0.06



a) What was the initial height of the ball? 2m

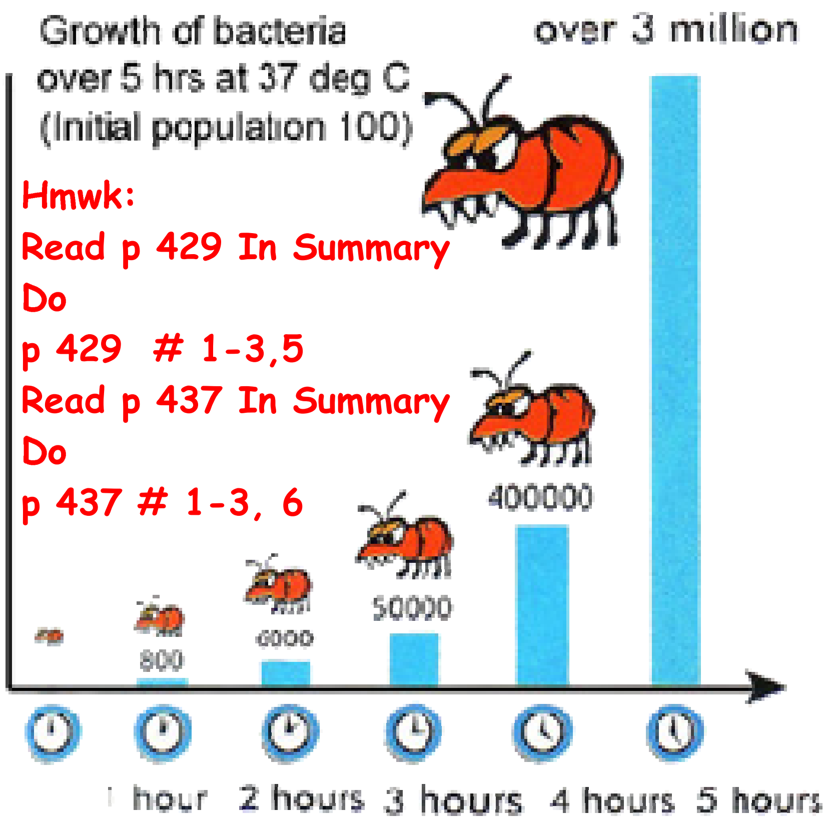
b) What was the height of the ball after 1 bounce? 1.2m after 2 bounces? 0.72m

after 3 bounces? ____________ after 4 bounces ____________

c) At what rate is the height decreasing? 60%/bounce

d) Looking at your graph approximately when will your ball stop bouncing? ~10 bounces

d) According to your equation will your ball ever stop bouncing? no
(hint try 25 bounces). Explain.



Hmwk:
Read p 429 In Summary
Do
p 429 # 1-3,5
Read p 437 In Summary
Do
p 437 # 1-3, 6