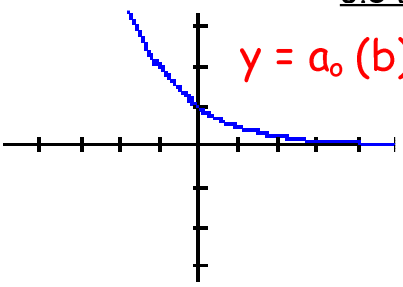
6.8 Exponential Decay

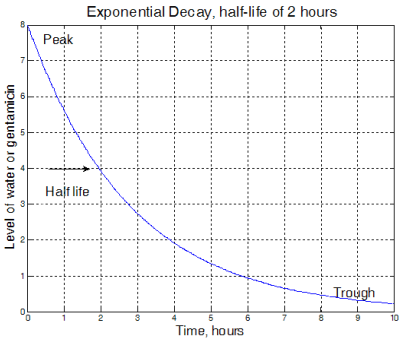


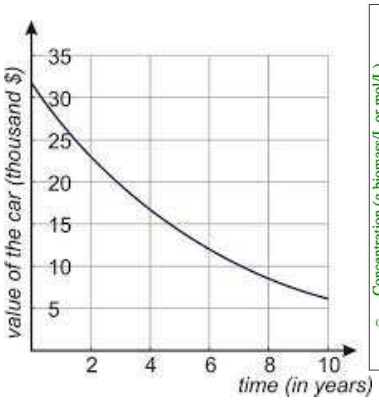
$y = a_o (b)^x$

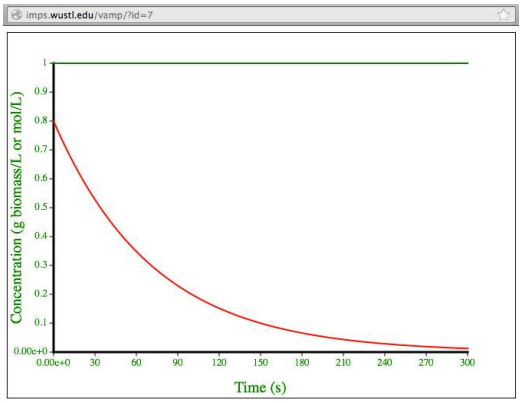
Seen in:

- DEPRECIATION on an automobile
- DEPRECIATION in the value of a home
- Population DECAY
- **HALF-LIFE** of a radioactive element

(time it takes for 1/2 the population to decay)

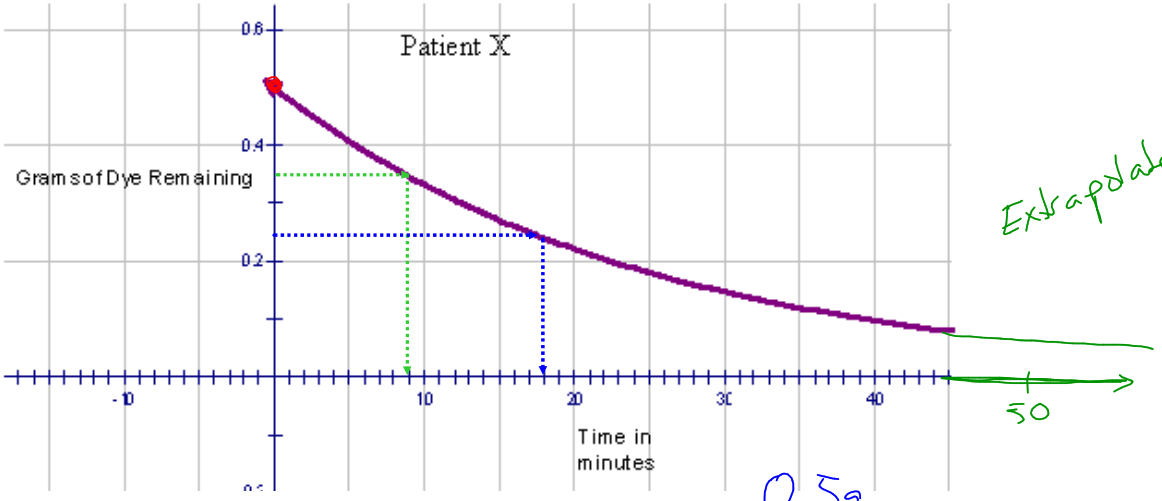






Ex 1:

Dye is injected into a patient to test pancreas function. The mass, R grams of dye remaining in a healthy pancreas after t minutes is given by the equation $R = M(0.96)^t$, where M grams is the mass of dye initially injected. The graph below represents the results of a patient having their pancreas tested:



a) How many grams of dye were injected initially into the patient? 0.5g

Using the graph

b) How much time has elapsed when 0.35 g remains? approx 9 min

c) How long does it take for the patient to have half the amount of dye in their system? Approx. 18 min
(this is half life)

d) How many grams of dye remain after 50 minutes? approx 0.075

e) Use the equation above to verify your answer in part c). (Show your substitution) and d)

c) $R = 0.5(0.96)^{18}$
$= 0.24$

hmmm... is $t = 17$ closer?

$R = 0.5(0.96)^{17}$
$= 0.25$

∴ 17 min is more accurate

d) $R = 0.5(0.96)^{50}$
$\doteq 0.065$

We were close!

Formula: $P(n)=P_o(1-r)^n$

P(n): final amount
r: rate of decay
P_0: initial amount
n: number of decay periods

When the base (1-r) is $0<(1-r)<1$: Decay

Ex 2: page 438 # 3

A new car costs $24000. It loses 18% of its value each year after it is purchased. This type of loss is called depreciation. The value of the car is given by $V(n)=V_o(1-0.18)^n$, where V_0 is the original value of the car, and n is the number of years after the car was purchased.

a) Use the formula to determine how much of the car's initial value is lost after 5 years.

$V_n = 24000(0.82)^5$
$\doteq 8897.76$

Value lost = Initial - Current Price
$= 24000 - 8897.76$
$\doteq 15102.24$

$\therefore$ Value lost is $15102.24

b) Use the formula to determine the value of the car after 30 months.

This equation is for years! $\therefore$ 30 months $= \frac{30}{12} = 2.5$ yrs

$V(2.5) = 24000(0.82)^{2.5}$
$= 14613.22$

$\therefore$ The value would be $14613.22

Ex 3: p 438 # 7

Gels used to change the colour of spotlights each reduce the intensity of the light. The algebraic model for this situation is $I=100(0.96)^n$

a) Describe what each part of the equation represents.

100 ↑ Original Intensity of light (100%)

0.96 Represents 4% decay in intensity caused by the gel

n # of gels

b) State the rate of decay.

4%

$1 - r = 0.96$

$r = 0.04$

c)Determine the intensity of the spotlight if three gels are used.

$I = 100(0.96)^3$

$= 89$

∴ The light intensity is approx. 89%

d) How many gels would reduce the intensity by more than 75%?

Guess & Check

meaning down to 25%

$n = 5 \Rightarrow 100(0.96)^5 = 81.5$

$n = 7 \Rightarrow 100(0.96)^7 = 75.145$

⋮

$n = 21 \Rightarrow 100(0.96)^{21} \doteq 42$

$n = 27 \Rightarrow 100(0.96)^{27} \doteq 33$

$n = 34 \Rightarrow 100(0.96)^{34} \doteq 24.95$

∴ We would need 34 gels!!!

Ex 4: p 439 # 9

A hot cup of coffee cools accordingly to the equation $T(t) = 68\left(\frac{1}{2}\right)^{\frac{t}{22}}$ where T(t) is the temperature in degrees Celsius, and t is time in minutes.

a) Which part of the equation indicates that it models exponential decay?

The base of $\frac{1}{2}$ or 0.5

b) What value of t makes the exponent in the equation equal to 1?

22

c) What is the significance of this value?

We will have multiplied by $\frac{1}{2}$ ONCE after 22 minutes.

Half-life

d)What was the initial temperature of the coffee?

68° C

e) determine the temperature of the coffee after 40 minutes

$T(40) = 68\left(\frac{1}{2}\right)^{\frac{40}{22}}$

$\doteq 19.28°$

∴ The temp is approx 19°

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After... 4 hours? 240 minutes?

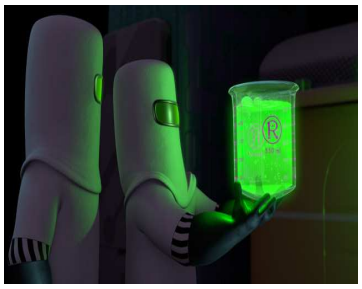
$T(240) = 0.03...$?

Try → $T(t) = 68\left(\frac{1}{2}\right)^{\frac{n}{22}} + 21$

Room temp!

Moves everything up 21°

Ex. 5: The half-life of a radioactive element is 15 days. This means that every 15 days, the amount decreases by 50%. How much of a 200 gram sample will be left after 150 days?



$(\frac{1}{2})^{\frac{t}{h}}$

$h = 15$

$A_0 = 200$

$t = 150$

$A = A_0(\frac{1}{2})^{\frac{t}{h}}$

$= 200(\frac{1}{2})^{\frac{150}{15}}$

$\doteq 0.1953$

$\therefore$ Approx 0.2g after 150 days

Hmwk:
p 437 # 5, 8, 10 - 13