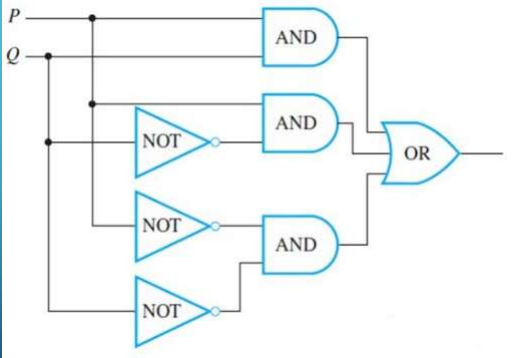


# BOOLEAN IDENTITIES

1

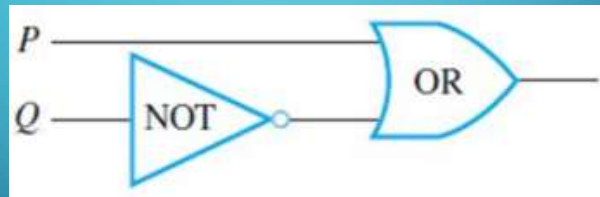
- Complete the truth table and find the Boolean Expression for the output



The diagram shows a logic circuit with two inputs,  $P$  and  $Q$ . The circuit consists of three AND gates and three NOT gates. The first AND gate has inputs  $P$  and  $Q$ . The second AND gate has inputs  $P$  and the output of a NOT gate connected to  $Q$ . The third AND gate has inputs the output of a NOT gate connected to  $P$  and the output of a NOT gate connected to  $Q$ . The outputs of these three AND gates are connected to an OR gate.

2

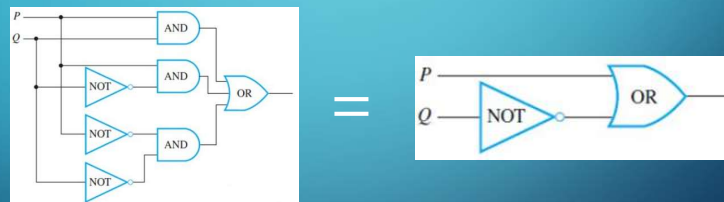
- Complete the truth table and find the Boolean Expression for the output



3

## WHY DO WE NEED BOOLEAN ALGEBRA?

- Boolean algebra allows for the simplification of complex logic circuits



4

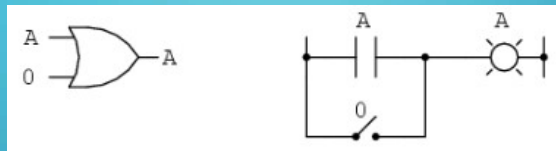
## PROVING EQUIVALENCE

- Method 1: Proof by perfect induction (brute force)
  - Compare the complete truth tables of both circuits
- Method 2: Boolean Identities
  - Compare LS to RS

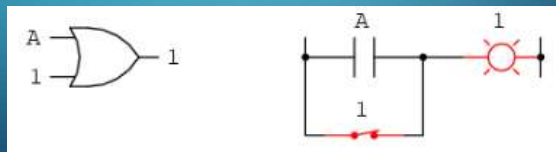
5

## BOOLEAN IDENTITIES

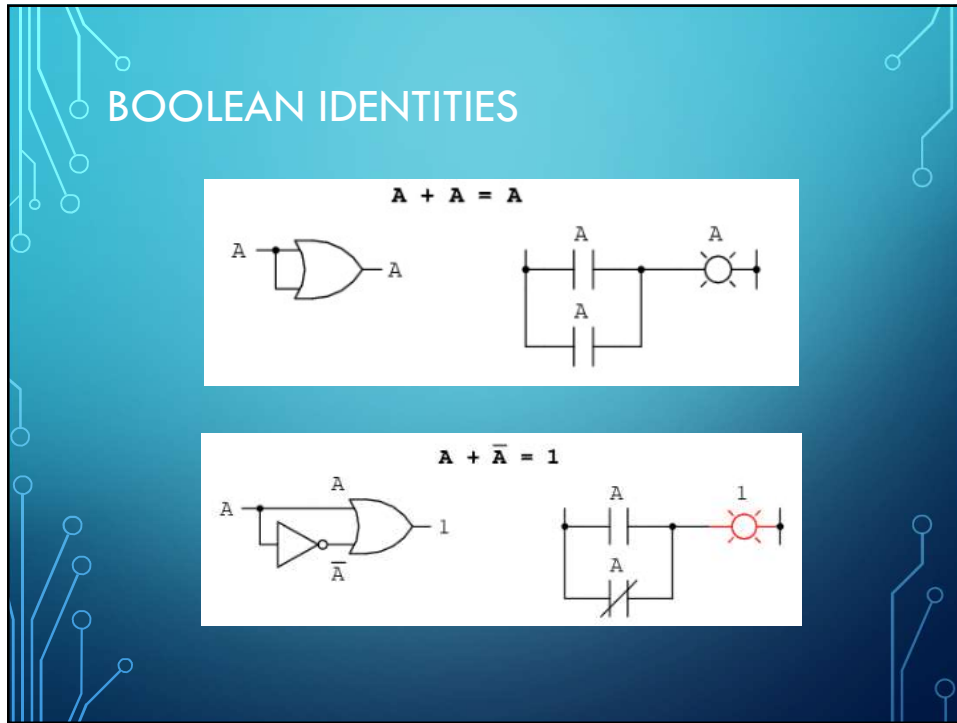
- $A+0=A$



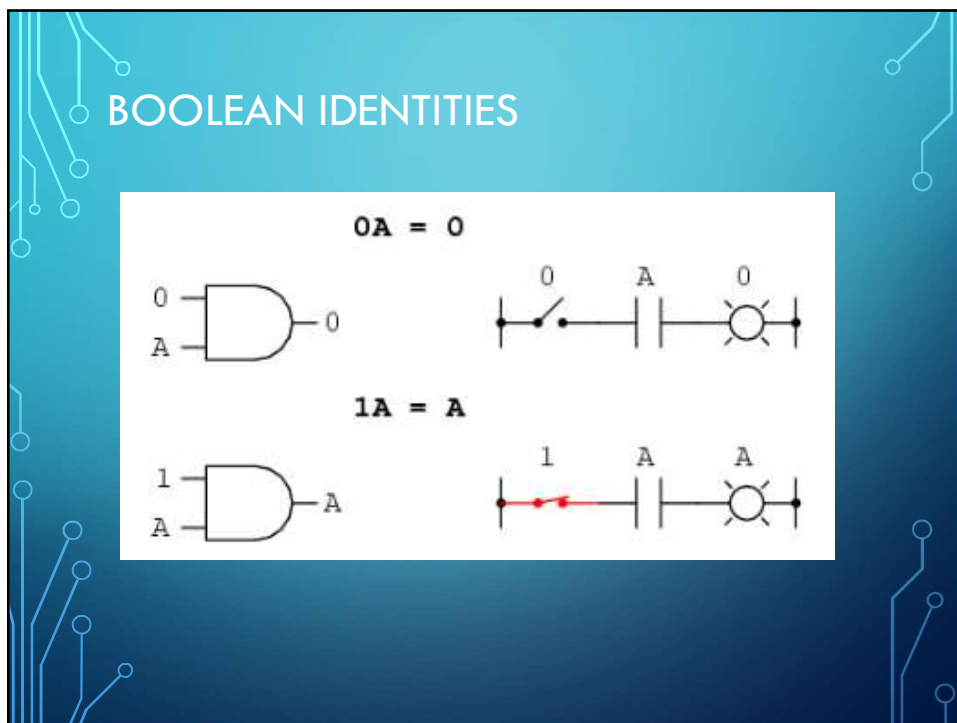
- $A+1=1$



6



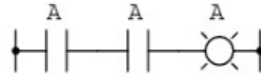
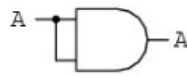
7



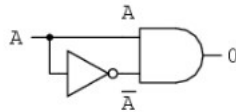
8

## BOOLEAN IDENTITIES

$$AA = A$$



$$A\bar{A} = 0$$



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## SUMMARY OF IDENTITIES

### Additive

$$A + 0 = A$$

$$A + 1 = 1$$

$$A + A = A$$

$$A + \bar{A} = 1$$

### Multiplicative

$$0A = 0$$

$$1A = A$$

$$AA = A$$

$$A\bar{A} = 0$$

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## COMMUTATIVE PROPERTY

- Addition
  - $A+B=B+A$

- Multiplication
  - $AB=BA$

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## ASSOCIATIVE PROPERTY

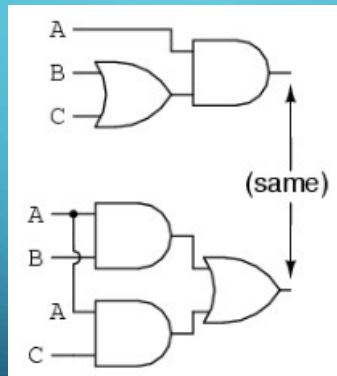
- Addition
  - $A+(B+C)=(A+B)+C$

- Multiplication
  - $A(BC)=(AB)C$

12

## DISTRIBUTIVE PROPERTY

- $A(B+C) = AB+AC$



13

## INVOLUTION PROPERTY

- $\overline{\overline{A}} = A$

## DEMORGAN'S THEOREM

- $\overline{A+B} = \overline{A} \cdot \overline{B}$
- $\overline{AB} = \overline{A} + \overline{B}$

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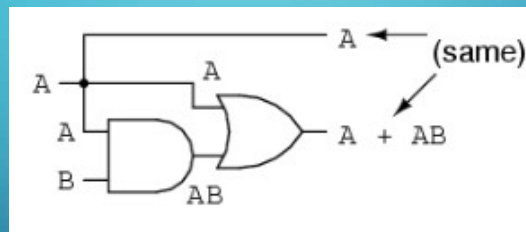
## SUMMARY

- $AA = A$
- $A0 = 0$
- $A1 = A$
- $A\bar{A} = 0$
- $A + A = A$
- $A + 0 = A$
- $A + 1 = 1$
- $A + \bar{A} = 1$
- $AB = BA$
- $A + B = B + A$
- $A + (B + C) = (A + B) + C$
- $(AB)C = A(BC)$
- $A(B + C) = AB + AC$
- $\bar{\bar{A}} = A$
- $\overline{AB} = \bar{A} + \bar{B}$
- $\overline{A + B} = \bar{A} \cdot \bar{B}$

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## BOOLEAN SIMPLIFICATIONS

- $A + AB = A$

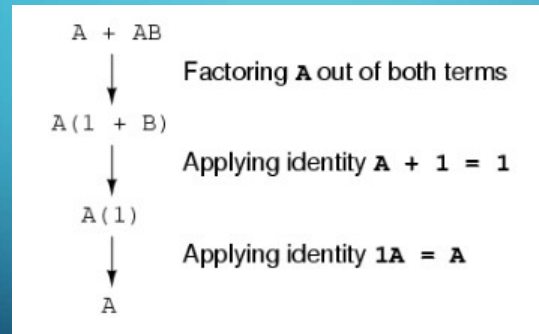


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## BOOLEAN SIMPLIFICATIONS

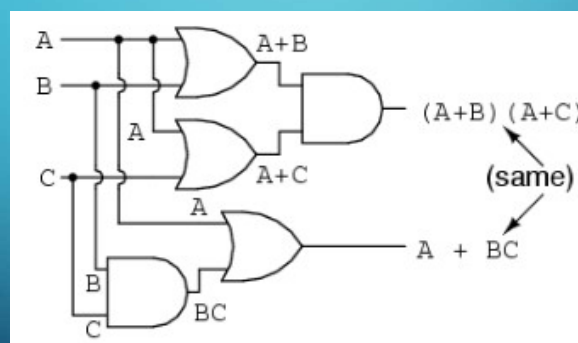
- $A + AB = A$



17

## BOOLEAN SIMPLIFICATIONS

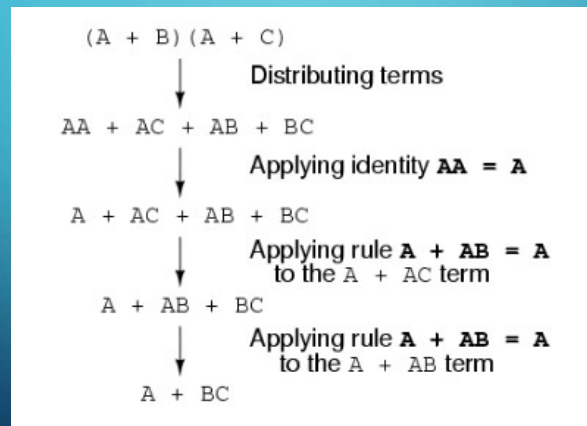
- $(A + B)(A + C) = A + BC$



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## BOOLEAN SIMPLIFICATIONS

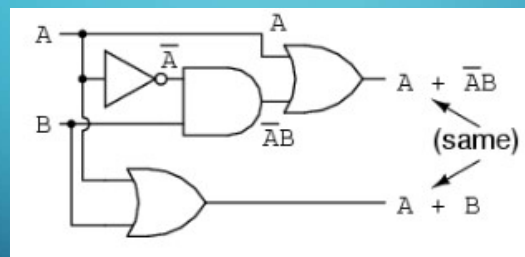
- $(A + B)(A + C) = A + BC$



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## BOOLEAN SIMPLIFICATIONS

- $A + \bar{A}B = A + B$



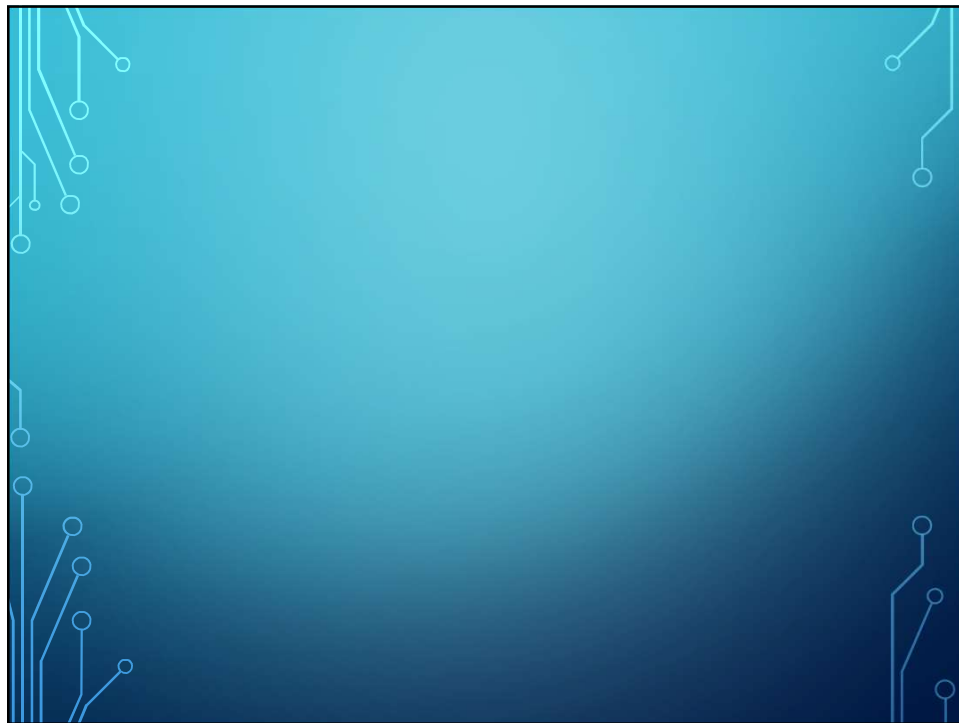
20

## BOOLEAN SIMPLIFICATIONS

- $A + \bar{A}B = A + B$

$$\begin{array}{l}
 A + \bar{A}B \\
 \downarrow \text{Applying the previous rule to expand } \mathbf{A} \text{ term} \\
 \mathbf{A + AB = A} \\
 A + AB + \bar{A}B \\
 \downarrow \text{Factoring } \mathbf{B} \text{ out of 2}^{\text{nd}} \text{ and 3}^{\text{rd}} \text{ terms} \\
 A + B(A + \bar{A}) \\
 \downarrow \text{Applying identity } \mathbf{A + \bar{A} = 1} \\
 A + B(1) \\
 \downarrow \text{Applying identity } \mathbf{1A = A} \\
 A + B
 \end{array}$$

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