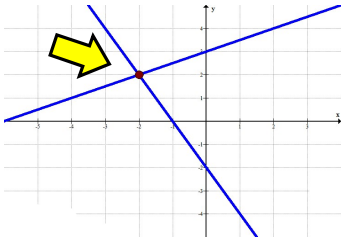
2.3 Solving Systems by Substitution

Recall: Solving a system of equations means...

Finding the point of intersection of the lines.

It is the only point where both lines have the same x-value and y-value.



Drawbacks of solving by graphing...

- not always accurate
- time consuming

What's so great about solving algebraically?

- gives exact values
- less time / less space

Ex. 1 How would you solve

$x = 5$ (1)

$3x - 4y = 3$ (2)

Sub (1) into (2)

$3x - 4y = 3$

$3(5) - 4y = 3$

$15 - 4y = 3$

$-4y = 3 - 15$

$-4y = -12$

$y = 3$

∴ Intersection is $(5, 3)$

Ex. 2 Rogers charges $35/month plus $10 for every extra Gb. Bell charges $40/month plus $8 for every extra Gb. When are they the same monthly price? Solve without graphing.

Let C be the monthly cost

Let g be # of extra Gb

Rogers $C = 35 + 10g$ (1)

Bell $C = 40 + 8g$ (2)

Sub (1) into (2)

$35 + 10g = 40 + 8g$

$10g - 8g = 40 - 35$

$2g = 5$

$g = 2.5$

∴ they have the same price when they use 2.5 GB

GRAPH

THE SUBSTITUTION METHOD:

1. **Isolate** a variable in one equation (pick the best one)
2. **Substitute** to create an equation with only one variable.
3. **Solve** the equation.
4. ***Substitute*** the solved variable into the equation from #1 to determine the value of the other variable.
5. **Write** a conclusion.
6. (**Check**) - formal if asked, otherwise complete a mental check.

Ex. 3 Solve using the substitution method.

a) $x + 3y = -4$ ①
$2x - 3y = 1$ ②

① $x + 3y = -4$

③ $x = -4 - 3y$

sub ③ into ②

$2x - 3y = 1$

$2(-4-3y) - 3y = 1$

$-8 - 6y - 3y = 1$

$-9y = 9$

$y = -1$

Sub $y = -1$ into ③

$x = -4 - 3y$

$x = -4 - 3(-1)$

$x = -4 + 3$

$x = -1$

$\therefore (-1, -1)$

b) $5a + 3b = 10$ ①
$2a - b = 4$ ②

② $2a - b = 4$

$2a = 4 + b$

$2a - 4 = b$ ③

Sub ③ into ①

$5a + 3(2a-4) = 10$

$5a + 6a - 12 = 10$

$11a = 22$

$a = 2$

Sub $a = 2$ into ②

$2(2) - b = 4$

$4 - b = 4$

$-b = 0$

$b = 0$

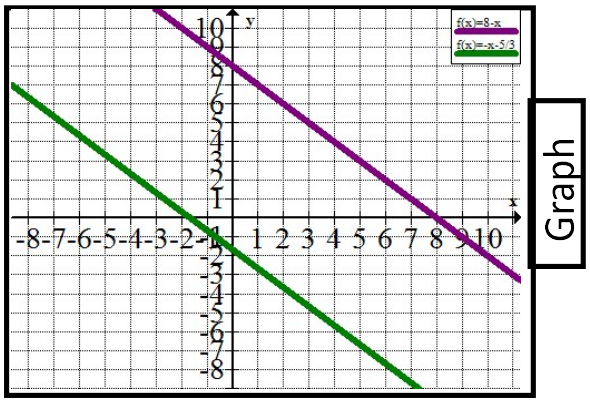
$\therefore$ Solution is $a = 2, b = 0$

or $(a, b) \Rightarrow (2, 0)$

SPECIAL CASES

A.

$$x + y = 8$$
$$3x + 3y = -5$$



Solution is $24 = -5$???

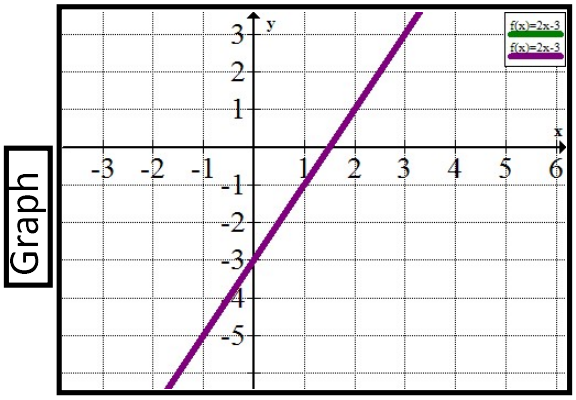
∴ No solutions!
(no intersections)

B.

$$p - 2q = -3$$
$$4q = 2p + 6$$

$4q = 4q$???

∴ Infinite number of solutions!
(Same line!)



Working with fractions...
pg. 26 #4a)

$x + 2y = 3$ ①
$5x + 4y = 8$ ②

① $x + 2y = 3$
$x = 3 - 2y$ ③

Sub ③ into ②
$5(3-2y) + 4y = 8$
$15 - 10y + 4y = 8$
$15 - 6y = 8$
$-6y = -7$
$y = \frac{7}{6}$

Sub $y = \frac{7}{6}$ into ①
$x + 2\left(\frac{7}{6}\right) = 3$
$x + \frac{7}{3} = 3$
$x = 3 - \frac{7}{3}$
$x = \frac{9}{3} - \frac{7}{3}$
$x = \frac{2}{3}$

∴ Solution is $\left(\frac{2}{3}, \frac{7}{6}\right)$

Challenge:

The following three lines all intersect at one point. List the steps you would follow to determine the coordinates of the point of intersection and the value of *k.*

$$2x + 3y = 7$$
$$x + 4y = 16$$
$$4x - ky = 9$$

2022 Homework:

Set 1: p.26 #1ab,2d,4bd,5ac,12

Set 2: p.26 #5de,12,17a,19,20b