6.1 The Equation of a Line

To write the equation of a line you need slope and y-intercept.

Recall: Rate of change = $\frac{\text{rise}}{\text{run}}$

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

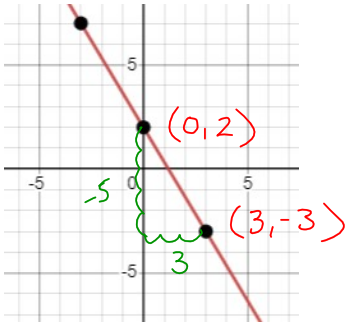
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$y = mx + b$

*slope = rate of change

Ex 1: Determine the slope for each of the following.

a)



$m = \frac{\text{rise}}{\text{run}}$

$= \frac{-5}{3}$

(0,2)
(3,-3)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3-2}{3-0}$

$= \frac{-5}{3}$

b) the line passing through point A(-7,6) and B(21,-10)

(-7,6)
(21,-10)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

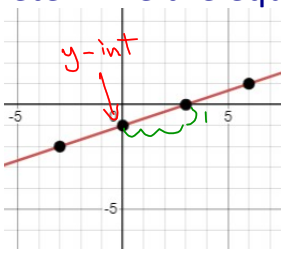
$= \frac{-10-6}{21-(-7)}$

$= \frac{-16}{28}$

$= \frac{-4}{7}$

Ex 2: Determine the equation for each of the following lines.

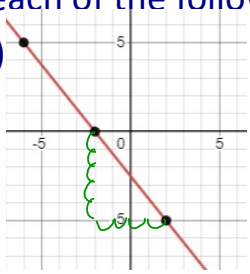
a)



$m = \frac{1}{3}$ $y = \frac{1}{3}x - 1$

y-int $= -1$

b)



*algebraically determine the value of "b"

$m = \frac{-5}{4}$

Set up generally $y = -\frac{5}{4}x + b$

sub in $(-6,5)$

$5 = -\frac{5}{4_2}(-6^3) + b$

$5 = +\frac{15}{2} + b$

$\frac{10}{2} - \frac{15}{2} = b$

$-\frac{5}{2} = b$

$\therefore y = -\frac{5}{4}x - \frac{5}{2}$

c) the line passing through points A(10,3) and B(-4,10)

$(10,3)$
$(-4,10)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{10-3}{-4-10}$

$= \frac{7}{-14}$

$= -\frac{1}{2}$

$y = -\frac{1}{2}x + b$

$3 = -\frac{1}{2}(10) + b$

$3 = -5 + b$

$8 = b$

$\therefore y = -\frac{1}{2}x + 8$

d) the line passing through points C(3,-4) and D(-1,7)

$(3,-4)$
$(-1,7)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-(-4)}{-1-3}$

$= \frac{11}{-4}$

$= -\frac{11}{4}$

$y = -\frac{11}{4}x + b$

Sub in $(3,-4)$

$-4 = -\frac{11}{4}(3) + b$

$-4 = -\frac{33}{4} + b$

$-\frac{16}{4} + \frac{33}{4} = b$

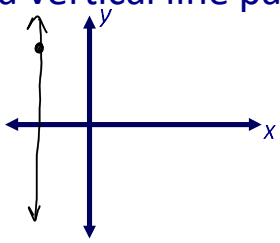
$\frac{17}{4} = b$

$\therefore y = -\frac{11}{4}x + \frac{17}{4}$

SPECIAL CASES: Horizontal & Vertical Lines

Ex 3: Determine the equation for each of the following lines.

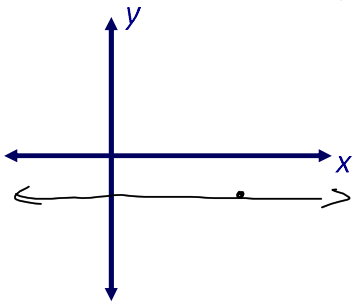
a) a vertical line passing through (-3,5)



the x-coordinate is always -3

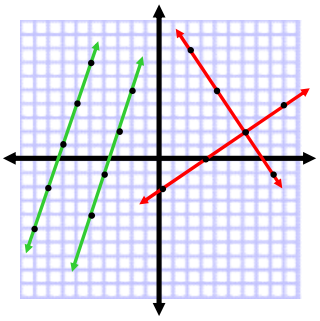
$x = -3$

b) a horizontal line passing through (7,-2)



the y-coordinate is always -2

$y = -2$



Note:

Parallel lines have the same slope.

Perpendicular lines have slopes that are negative reciprocals.

Ex 4 Determine the equation of a line :

a) parallel to 9x - 3y = 12 and passing through the point D(3,5).

Put in $y=mx+b$ form to find slope!

$9x-3y=12$

$9x-12=3y$

$3y=9x-12$

$y=3x-4$

slope! $m=3$

$y=3x+b$

sub (3,5)

$5=3(3)+b$

$5=9+b$

$-4=b$

$\therefore y=3x-4$

b) perpendicular to y = -2x +7 and passing through the point D(2,-3).

$m=-2$

Perp. slope $m=\frac{1}{2}$

$y=\frac{1}{2}x+b$

sub in (2,-3)

$-3=\frac{1}{2}(2)+b$

$-3=1+b$

$-4=b$

$\therefore y=\frac{1}{2}x-4$

c) perpendicular to 4x +3y - 7 = 0 and has the same x-intercept as 2x + 3y - 12 = 0

Find slope from $4x+3y-7=0$

$3y=-4x+7$

$y=-\frac{4}{3}x+\frac{7}{3}$

$m=-\frac{4}{3}$

Perp. slope $m=\frac{3}{4}$

Find x-int

sub $y=0$

$2x+3(0)-12=0$

$2x=12$

$x=6$

x-int (6,0)

$y=\frac{3}{4}x+b$

sub in (6,0)

$0=\frac{3}{4}(6)+b$

$0=\frac{9}{2}+b$

$b=-\frac{9}{2}$

$\therefore y=\frac{3}{4}x-\frac{9}{2}$