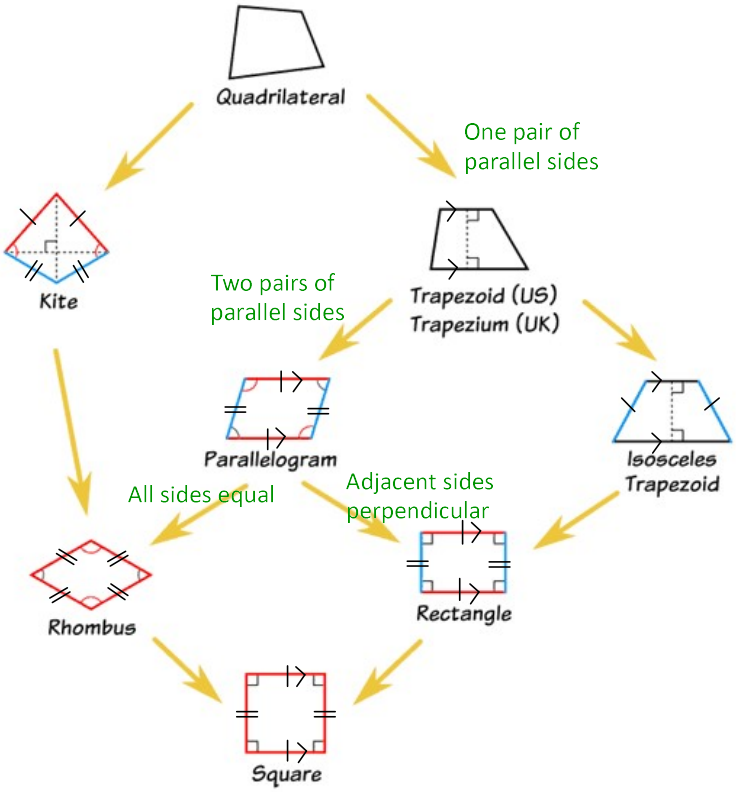
6.5 Verify Properties of Quadrilaterals

Classifying Quadrilaterals:



Using your formulas for slope, midpoint, and distance,what would you have to do to prove that a quadrilateral is a.....

kite - lengths of all 4 sides, pairs of adjacent sides are equal

trapezoid -slopes of one pair of opposite sides must be equal
For isosceles trapezoid...check that 2 non-parallel sides have equal length

parallelogram -slopes of opposite sides must be equal

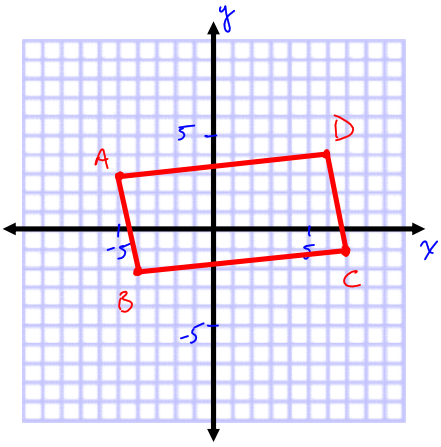
rhombus - use distance formula to find all 4 lengths...they must all be equal

square - use distance formula to find all 4 lengths...they must all be equal
- use slope formula to show that we have 2 pairs of equal slopes that are negative reciprocals

rectangle - use distance formula to find all 4 lengths...opposite sides must be equal
- use slope formula to show that we have 2 pairs of equal slopes that are negative reciprocals

Ex 1: Classify quadrilateral ABCD defined by: A(-5,3), B(-4,-2), C (7,-1), and D(6,4).

Note: Make sure you connect the vertices in order....A is attached to B...B to C, C to D and D to A but they do not always need to be in alphabetical order.



$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-3}{-4-(-5)}$

$= \frac{-5}{1}$

$= -5$

$m_{BC} = \frac{1}{11}$

$m_{CD} = -5$

$m_{DA} = \frac{1}{11}$

$m_{AB} \parallel m_{CD}$

$m_{BC} \parallel m_{DA}$

(parallel)

GRAPH

Length AB

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5-(-4))^2 + (3-(-2))^2}$

$= \sqrt{1 + 25}$

$= \sqrt{26}$

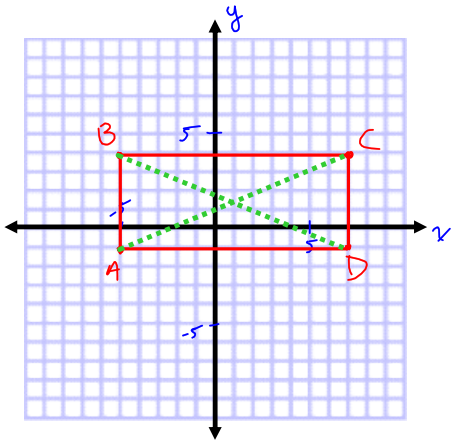
Length BC

$d = \sqrt{122}$

- Two parallel sides
- Not perpendicular (slopes not negative reciprocals)
- Sides not equal

This is a parallelogram!

Ex 2: Given rectangle A(-5,-1), B(-5,4), C(7,4) and D(7,-1) draw a sketch of the quadrilateral. Prove that diagonals AC and BD bisect each other.



Do they have the same midpoint?

$MP_{AC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-5+7}{2}, \frac{-1+4}{2}\right)$

$= \left(1, \frac{3}{2}\right)$

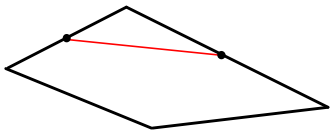
$MP_{BD} = \left(1, \frac{3}{2}\right)$

$MP_{AC} = MP_{BD}$

∴ Yes! They bisect each other

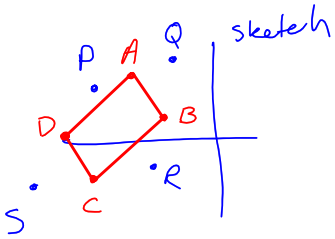
GRAPH

Midsegments: line segments that connect midpoints of adjacent sides.



Ex 3: Show that the midsegments of this quadrilateral with vertices P(-5,2), Q(-1,3), R(-2,-1) and S(-6,-2) form a rectangle.

① Find the midpoints
② Find slopes



GRAPH

① $MP_{PQ} = \left(\frac{-5+(-1)}{2}, \frac{2+3}{2}\right)$ $MP_{RS} = \left(-4, -\frac{3}{2}\right)$

$A = \left(-3, \frac{5}{2}\right)$ $C = \left(-4, -\frac{3}{2}\right)$

$MP_{QR} = \left(-\frac{3}{2}, 1\right)$ $MP_{SP} = \left(-\frac{11}{2}, 0\right)$

$B = \left(-\frac{3}{2}, 1\right)$ $D = \left(-\frac{11}{2}, 0\right)$

② Find Slopes

$m_{AB} = \frac{1-\frac{5}{2}}{-\frac{3}{2}-(-3)} = \frac{\frac{2}{2}-\frac{5}{2}}{-\frac{3}{2}+\frac{6}{2}} = \frac{-\frac{3}{2}}{\frac{3}{2}} = -1$

$m_{BC} = \frac{-\frac{3}{2}-1}{-4-(-\frac{3}{2})} = \frac{-\frac{5}{2}}{-\frac{8}{2}+\frac{3}{2}} = \frac{-\frac{5}{2}}{-\frac{5}{2}} = 1$

$m_{CD} = \frac{-\frac{3}{2}-0}{-4-(-\frac{11}{2})} = \frac{-\frac{3}{2}}{-\frac{8}{2}+\frac{11}{2}} = \frac{-\frac{3}{2}}{\frac{3}{2}} = -1$

$m_{DA} = \frac{0-\frac{5}{2}}{-\frac{11}{2}-(-3)} = \frac{-\frac{5}{2}}{-\frac{11}{2}+\frac{6}{2}} = \frac{-\frac{5}{2}}{-\frac{5}{2}} = 1$

- Two sets of parallel lines
- Are negative reciprocals

Yes! This is a rectangle