3.7 Applications of Exponential Functions

Recall:

Amount after "x" growth/decay periods

# of growth/decay periods

$$A = a_o\,(b)^x$$

Amount at beginning.

growth factor (b > 1)
decay factor (0< b < 1)

Note: To calculate the number of growth/decay periods, divide the *total time* by the *Doubling* or *Halving time* (i.e. the growth/decay time).

$$x = \frac{t}{d} \quad \text{or} \quad x = \frac{t}{h}$$

Ex. 1 A certain strain of yeast cell doubles under certain conditions every 20 minutes. If there were 350 cells initially, how many will there be in 3 hours?

Given
$a_o = 350$
×2 every 20min
A @ 3hrs?
3 hrs = 180 min

$A = 350(2)^{\frac{x}{20}}$

Let x be # minutes

$= 350(2)^{\frac{180}{20}}$
$= 350(2)^9$
$= 179\ 200$

∴ There will be 179 200 cells

Ex. 2 The half-life of a radioactive element is 15 days.
a) Write a function relating the amount remaining, in grams, to the time, in days.
b) How much of a 200 gram sample will be left after 150 days?

Givens
$b = \frac{1}{2}$
a_o

a) $A = a_o\left(\frac{1}{2}\right)^{\frac{d}{15}}$

b) $A = 200\left(\frac{1}{2}\right)^{\frac{150}{15}}$
$= 200\left(\frac{1}{2}\right)^{10}$
$\doteq 0.195$

∴ There will be approx. 0.195g after 150 days

Ex. 3 In 2001, the population of Canada was 31 051 000. What is the percent growth, if the current population of Canada is 38 246 508 ?

assume 2022 is current year

$x = 2022 - 2001$
$= 21$

$$A = a_0 b^x$$
$$38246508 = 31\,051\,000 (b)^{21}$$
$$\sqrt[21]{\frac{38246508}{31\,051\,000}} = \sqrt[21]{b^{21}}$$
$$1.01 \doteq b$$

$\therefore$ The population grew by 1% each year.

Ex. 4 A radioactive substance has a half life of 2.4 days.

a) What fraction of the original amount would remain after 12 days?

b) How long would it take until only 12.5% of the original amount remained?

a)

$x = 12$

$$A = a_0 \left(\tfrac{1}{2}\right)^{\frac{x}{2.4}}$$
$$A = a_0 \left(\tfrac{1}{2}\right)^{\frac{12}{2.4}}$$
$$A = a_0 \left(\tfrac{1}{2}\right)^{5}$$
$$A = a_0 \left(\tfrac{1}{32}\right)$$

This is the remaining fraction! $\frac{1}{32}$

b) Use fake amounts to solve

ie. $a_0 = 100$
$A = 12.5$

$$12.5 = 100 \left(\tfrac{1}{2}\right)^{\frac{x}{2.4}}$$
$$\frac{12.5}{100} = \left(\tfrac{1}{2}\right)^{\frac{x}{2.4}}$$
$$\frac{1}{8} = \left(\tfrac{1}{2}\right)^{\frac{x}{2.4}}$$
$$\left(\tfrac{1}{2}\right)^{3} = \left(\tfrac{1}{2}\right)^{\frac{x}{2.4}}$$
$$\therefore\ 3 = \frac{x}{2.4}$$
$$x = 7.2$$

$\therefore$ It would take 7.2 days

Ex. 5 256g of a substance decays to 64g in 15.6 hours. Determine the half-life of the substance.

$\Lambda \ \lambda$

$$A = a_0\left(\frac{1}{2}\right)^{\frac{x}{t}}$$

Finding this!

$$64 = 256\left(\frac{1}{2}\right)^{\frac{15.6}{t}}$$

$$\frac{64}{256} = \left(\frac{1}{2}\right)^{\frac{15.6}{t}}$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{15.6}{t}}$$

$$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{\frac{15.6}{t}}$$

$$\therefore \quad 2 = \frac{15.6}{t}$$

$$t = \frac{15.6}{2}$$

$$= 7.8$$

$\therefore$ The half-life is 7.8 hours

HOMEWORK
Handout