Lesson 4.0: Review of Trigonometry

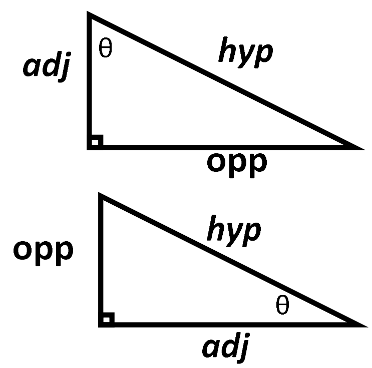
Recall: In a right triangle, the primary trig ratios are:

sine $\quad \sin\theta = \dfrac{\textbf{opp}}{\textbf{hyp}}$

cosine $\quad \cos\theta = \dfrac{\textbf{adj}}{\textbf{hyp}}$

tangent $\quad \tan\theta = \dfrac{\textbf{opp}}{\textbf{adj}}$

SOHCAHTOA

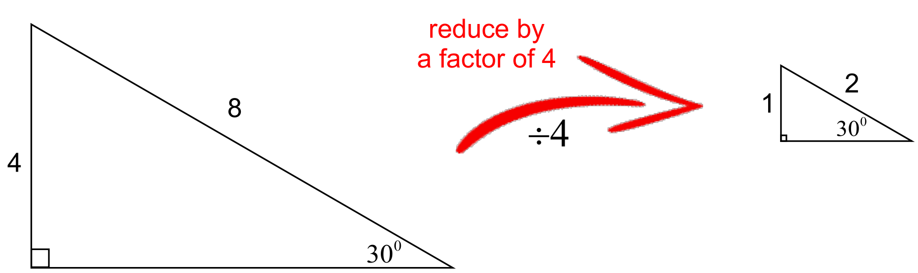


adj

opp

hyp

These ratios compare the lengths of the sides of a triangle. Trig stems from similar triangles. Any right triangle with a 30^0 angle (for example), whatever its size, will have the same ratio of sides lengths because the angles are the same!



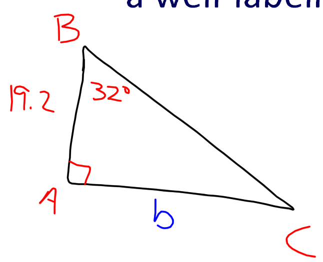
$\sin 30^0 = \frac{1}{2}$

$\sin 30^0 = \frac{4}{8}$

$= \frac{1}{2}$

Recall: To "solve a triangle" means to find the measures of all 3 sides and all 3 angles.

Ex. 1 In ΔABC, <A = 90°, <B = 32°, and c = 19.2 cm. Solve the triangle. Include a well-labelled diagram.



$\angle C = 180 - 90 - 32$
$= 58°$

$\underline{b}$

$\tan B = \frac{opp}{adj}$

$\tan 32° = \frac{b}{19.2}$

$19.2 \cdot \tan 32° = b$

$b \doteq 12$

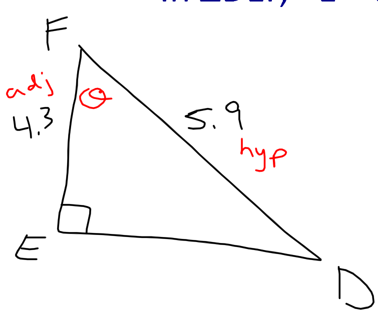
$\underline{a}$

$a^2 = b^2 + c^2$
$= 12^2 + 19.2^2$
$= 512.64$

$a = \sqrt{512.64}$
$\doteq 22.64$
$\doteq 22.6$

$\therefore$ a = 22.6 cm
b = 12 cm
$\angle C = 58°$

Ex. 2 In ΔDEF, <E = 90°, d = 4.3 m, and e = 5.9 m. Solve for $< F$.



CAH

$\cos\theta = \frac{adj}{hyp}$

$\cos\theta = \frac{4.3}{5.9}$

$\theta = \cos^{-1}\left(\frac{4.3}{5.9}\right)$

$\doteq 43°$

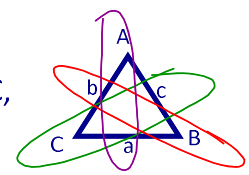
But what if the triangle is not right-angled?

Recall:

The Sine Law

Focus on side-angle pairs

In ΔABC,



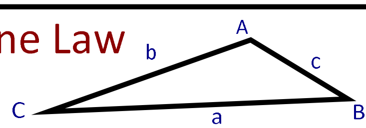
Finding side?

$$\frac{a}{sinA}=\frac{b}{\sin B}=\frac{c}{\sin C} \quad or \quad \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

Finding angle?

The Cosine Law

In ΔABC,



Two known sides + contained angle

$c^2 = a^2 + b^2 - 2ab \cos C$ rearrange --> $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

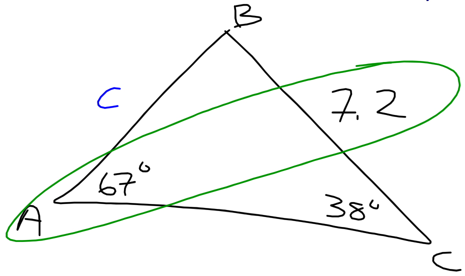
Need all side lengths

(Used when finding a side). (Used when finding an angle).

$b^2 = a^2 + c^2 - 2ac \cos B$

We will derive these formulas in lesson 4.4 A

Ex. 3 Solve for c in ΔABC, if <A = 67º, <C = 38º, and a = 7.2 cm.



USE SINE LAW!

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

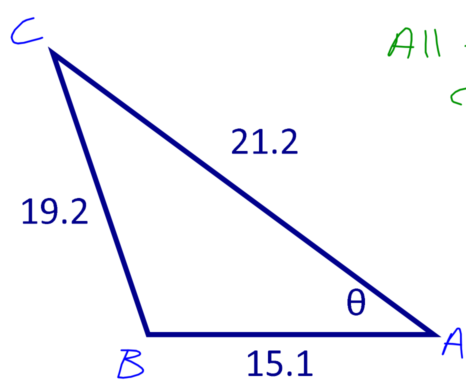
$$\frac{c}{\sin 38^\circ} = \frac{7.2}{\sin 67^\circ}$$

$$c = \sin 38^\circ \cdot \frac{7.2}{\sin 67^\circ}$$

$$\doteq 4.8$$

$\therefore c = 4.8\text{cm}$

Ex. 4 Solve for the unknown angle θ.



All three sides!
COSINE LAW

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$19.2^2 = 21.2^2 + 15.1^2 - 2(21.2)(15.1)\cos\theta$$

$$19.2^2 - 21.2^2 - 15.1^2 = -2(21.2)(15.1)\cos\theta$$

$$\frac{19.2^2 - 21.2^2 - 15.1^2}{-2(21.2)(15.1)} = \cos\theta$$

$$0.4823 \doteq \cos\theta$$

$$\theta = \cos^{-1}(0.4823)$$

$$\doteq 61.2^\circ$$

$\therefore A = 61.2^\circ$

p.220 #3a, 5b, 6c, 7, 8, 11-13