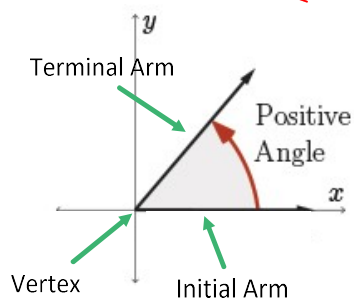


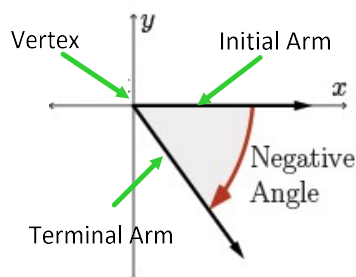
## Lesson 4.1: Angles between $0^\circ$ and $360^\circ$

An angle is in **standard position** when its vertex is at the origin and its initial arm is on the positive x-axis.

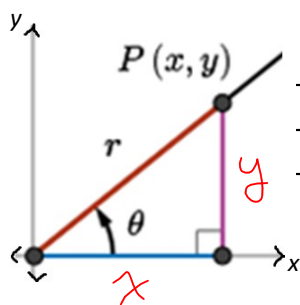
Counter Clockwise



Clockwise



Consider the point  $P(x,y)$  on the terminal arm of angle  $\theta$  in standard position.



The side **opposite** to  $\theta$  is  $y$ .  
The side **adjacent** to  $\theta$  is  $x$ .  
The **hypotenuse** is  $r$ .

SOH CAH TOA

$r^2 = x^2 + y^2$   
can find using  
Pythagorean Theorem

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

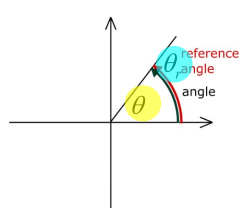
$$\tan \theta = \frac{y}{x}$$

**Principal Angle**

The angle measured between the initial arm and the terminal arm.

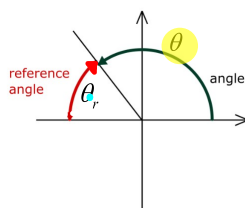
**Reference Angle or Related Acute Angle**

The angle between the terminal arm and the x-axis ( $\theta_r < 90^\circ$ ).



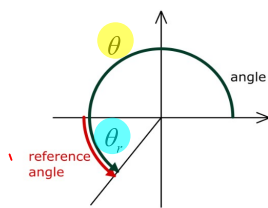
I quadrant

$$\theta_r = \theta$$



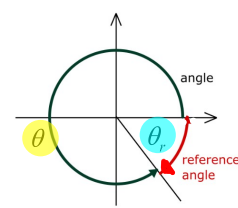
II quadrant

$$\theta_r = 180 - \theta$$



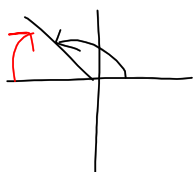
III quadrant

$$\theta_r = \theta - 180$$



IV quadrant

$$\theta_r = 360 - \theta$$



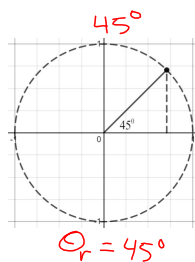
**Reference triangles  
are drawn to the x-axis.**

**Remember:  
your triangle should  
be part of a bowtie**



Exploring the primary trigonometric ratios on a coordinate grid.

**Terminal arm is in Quadrant 1**

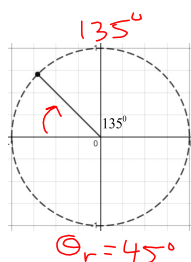


$$\sin \theta = 0.7071$$

$$\cos \theta = 0.7071$$

$$\tan \theta = 1$$

**Terminal arm is in Quadrant 2**

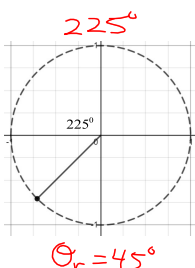


$$\sin \theta = 0.7071$$

$$\cos \theta = -0.7071$$

$$\tan \theta = -1$$

**Terminal arm is in Quadrant 3**

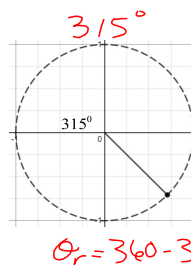


$$\sin \theta = -0.7071$$

$$\cos \theta = -0.7071$$

$$\tan \theta = 1$$

**Terminal arm is in Quadrant 4**



$$\sin \theta = -0.7071$$

$$\cos \theta = 0.7071$$

$$\tan \theta = -1$$

- For each of the above examples determine the reference/related acute angle,  $\theta_r$ .

- Evaluate

$$\sin 45 = 0.7071 \quad \sin 135 = 0.7071 \quad \sin 225 = -0.7071 \quad \sin 315 = -0.7071$$

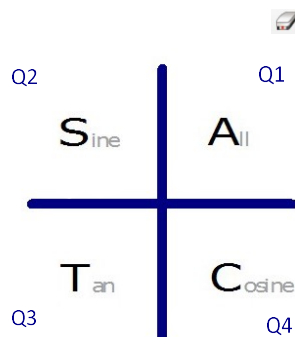
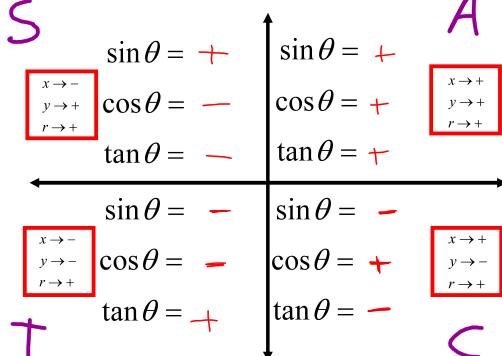
What do you notice?

The **Principal Angle** and the **Reference Angle** have the same trig ratios.

The only differences are with signs.

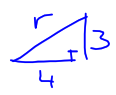


The CAST rule



Ex 1: Given the following points on the terminal arm of  $\theta$ , determine  $\sin \theta$ ,  $\cos \theta$ ,  $\tan \theta$ , and  $\theta$ .

a)  $P(4,3)$



$$r^2 = 4^2 + 3^2$$

$$r = \sqrt{25}$$

$$r = 5$$

$$x = 4$$

$$y = 3$$

$$r = 5$$

b)  $P(4,-3)$

$$r^2 = 4^2 + (-3)^2$$

$$r = \sqrt{25}$$

$$r = 5$$

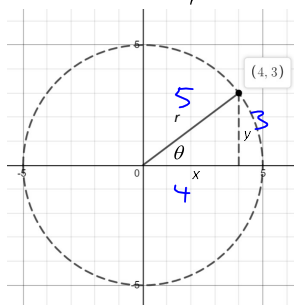
$$x = 4$$

$$y = -3$$

$$r = 5$$

Quadrant I

Note:  $\theta = \theta_r$



$$\sin \theta = \frac{y}{r} \rightarrow \frac{3}{5}$$

$$\cos \theta = \frac{x}{r} \rightarrow \frac{4}{5}$$

$$\tan \theta = \frac{y}{x} \rightarrow \frac{3}{4}$$

Solve for  $\theta$ :

$$\sin \theta = \frac{3}{5}$$

$$\theta_r = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\hat{=} 36.9^\circ$$

$$\theta = \theta_r$$

$$\therefore \theta = 36.9^\circ$$

$$\cos \theta = \frac{4}{5}$$

$$\theta_r = 36.9^\circ$$

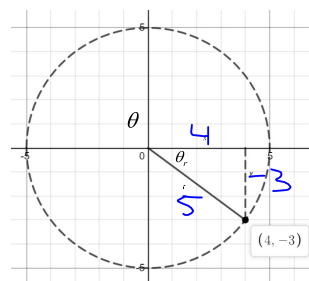
$$\theta = 360^\circ - \theta_r$$

$$\therefore \theta = 323.1^\circ$$

$$\sin \theta = -\frac{3}{5}$$

$$\theta = -36.9^\circ$$

Quadrant IV



$$\sin \theta = -\frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

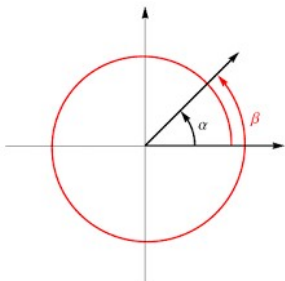
$$\tan \theta = -\frac{3}{4}$$

Solve for  $\theta$ :

Ex. 2: Evaluate, to four decimal places.

$$\text{a) } \cos 154^\circ \\ \doteq -0.8988$$

$$\text{b) } \tan 230^\circ \\ \doteq 1.1918$$

Co-terminal Angles:

- Angles with the same terminal arm.
- They can be positive or negative.

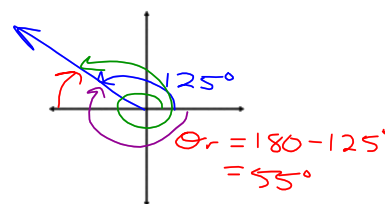
Ex. 3: Draw an angle of  $125^\circ$  in standard position.

$$\text{a) What is the principal angle? } \theta = 125^\circ$$

$$\text{b) What is the related acute angle? } \theta_r = 55^\circ$$

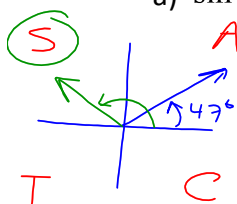
$$\text{c) Determine a positive angle that is co-terminal } \theta = \frac{360 + 125}{= 485^\circ}$$

$$\text{d) Determine a negative angle that is co-terminal } \theta = \frac{-(180 + 55)}{= -235^\circ}$$



Ex. 4: Determine another angle, with a different terminal arm, that would have the same trig ratios as the one given. Include a diagram.

$$\text{a) } \sin 47^\circ$$

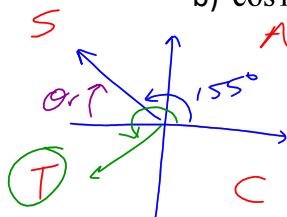


$$\theta_r = 47^\circ$$

Q2

$$\theta = 180 - 47^\circ \\ = 133^\circ$$

$$\text{b) } \cos 155^\circ$$



$$\theta_r = 180^\circ - 155^\circ \\ = 25^\circ$$

Q3

$$\theta = 180 + 25^\circ \\ = 205^\circ$$

NOTE: Q1 & Q2  
Sine was positive

p. 237 # 1ace, 3bcd, 4ace, 5, 11, 12a, 16