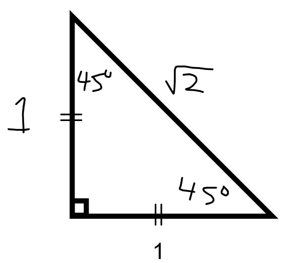
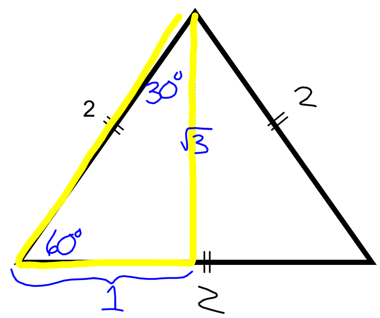
Lesson 4.2 : Special Angles and the Unit Circle

Consider the following triangles.
Determine the measure of all of the sides and the angles.





These are the Special Triangles:

45^0 (Right Isosceles Triangle)

$\sin 45^0 = \frac{1}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$

$\cos 45^0 = \frac{1}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$

$\tan 45^0 = \frac{1}{1} \rightarrow 1$

$30^0, 60^0$ (Half an Equilateral Triangle)

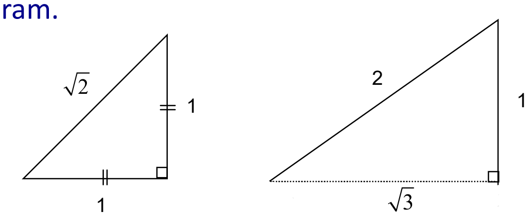
$\sin 30^0 = \frac{1}{2}$ $\quad$ $\sin 60^0 = \frac{\sqrt{3}}{2}$

$\cos 30^0 = \frac{\sqrt{3}}{2}$ $\quad$ $\cos 60^0 = \frac{1}{2}$

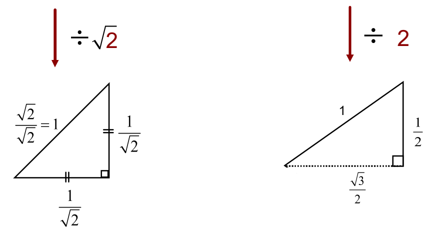
$\tan 30^0 = \frac{1}{\sqrt{3}} \rightarrow \frac{\sqrt{3}}{3}$ $\quad$ $\tan 60^0 = \frac{\sqrt{3}}{1} \rightarrow \sqrt{3}$

The Unit Circle: The unit circle is a way to "standardize" the ratios of the special angles onto one diagram.

The original special triangles:

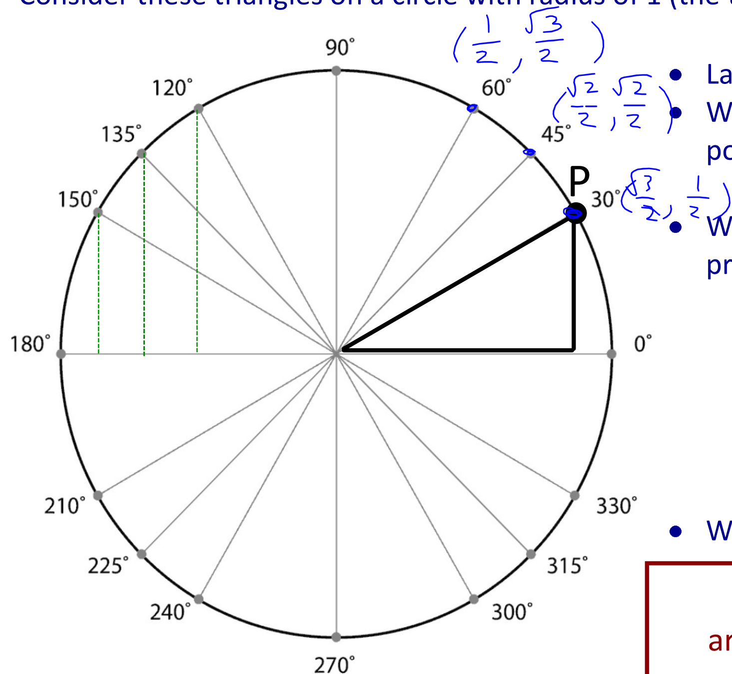


Now, make the hypotenuse 1:



Note: These are similar triangles to the original triangles.

Consider these triangles on a circle with radius of 1 (the terminal arm is 1 unit long).

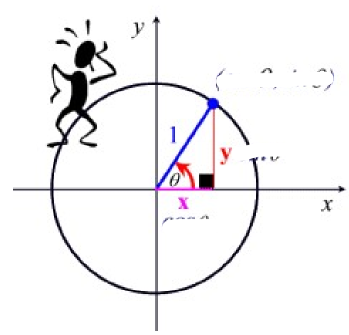


- Label the lengths of the sides.
- What are the coordinates of point P?
- What are the values of the primary trig ratios?
- What can you conclude?

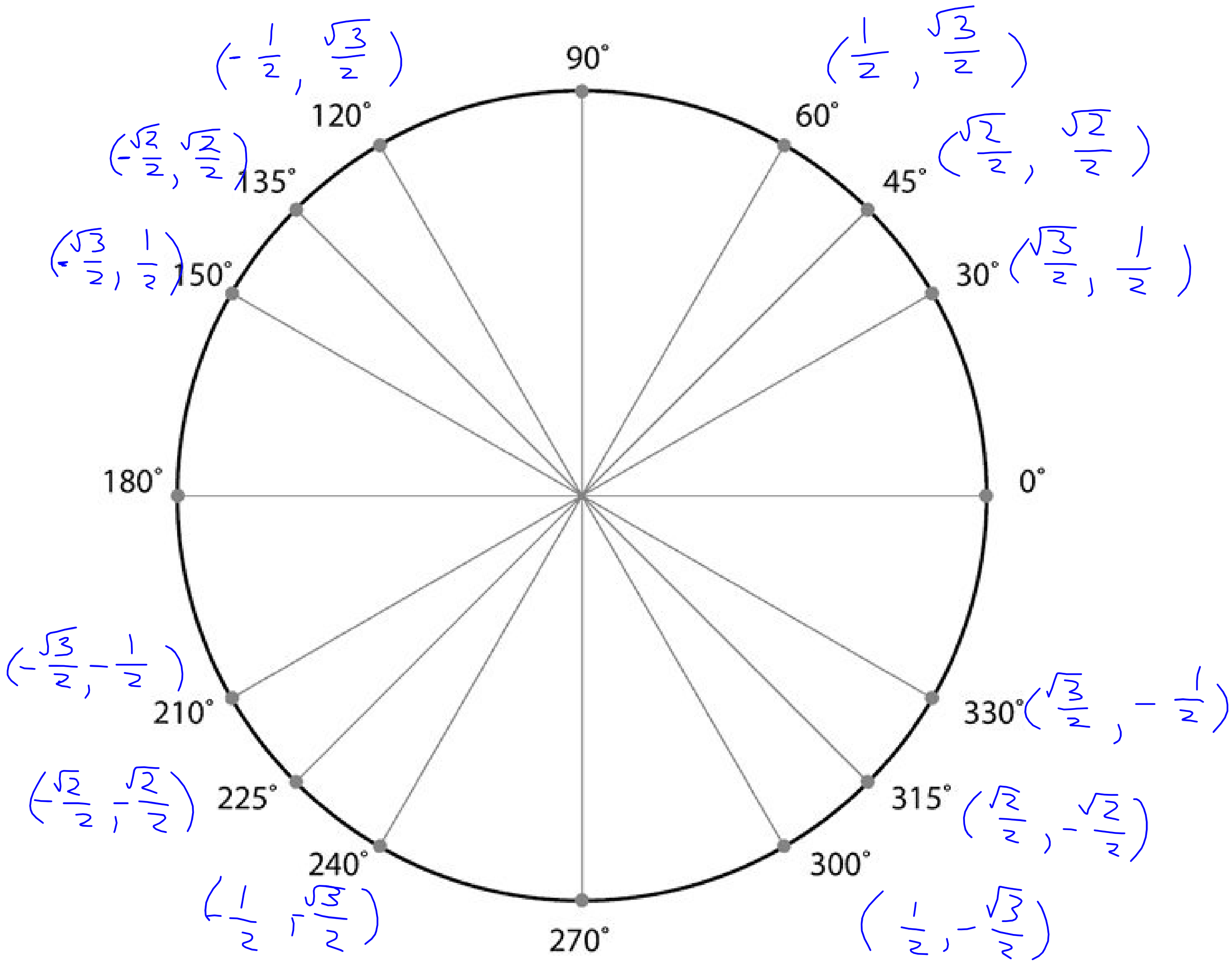
The coordinates are (c , si).

In general:

$$\cos\theta = \frac{x}{1} \qquad \sin\theta = \frac{y}{1} \qquad \tan\theta = \frac{y}{x}$$



Creating the Unit Circle

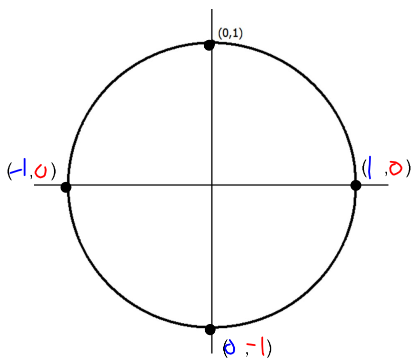


Think about the following...

- What happens to the y values as you rotate?
- What happens to the x values as you rotate?
- Connect the CAST rule to your knowledge of reflecting in the x-axis or y-axis.

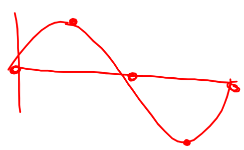
The unit circle allows us to understand the values of trig ratios for <u>axis angles</u>.

Terminal arm lies on the x-axis or y-axis

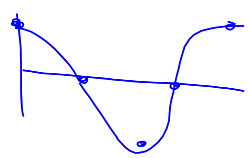


	0°	90°	180°	270°	360°
sin θ	0	1	0	−1	0
cos θ	1	0	−1	0	1
tan θ	$\frac{0}{1}=0$	$\frac{1}{0}$ = UNDEF	$\frac{0}{-1}=0$	$\frac{-1}{0}$ = UNDEF	$\frac{0}{1}=0$

sin θ



cos θ



Pull

Exact Values
NO ROUNDING
NO DECIMALS
NO CALCULATOR

Ex. 1 Determine the exact values.

a) $\cos 60^0$

$= \frac{1}{2}$

b) $\sin 45^0$

$= \frac{\sqrt{2}}{2}$

c) $\tan 30^0$

$= \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

$= \frac{1}{2} \div \frac{\sqrt{3}}{2}$

$= \frac{1}{2} \times \frac{2}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

d) $\sin 240^0$

$\theta_r = 240 - 180$ (Q3)

$= 60°$

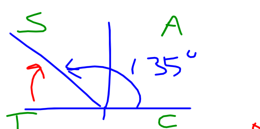
$\sin 240^0 = -\frac{\sqrt{3}}{2}$

Q3

Pull

Process
1. Determine the quadrant.
2. Diagram & find related angle.
3. Use special angles to write an equivalent ratio.
4. Use CAST rule to determine sign (+ or -).

e) $\tan 135^0$

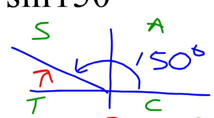


$\theta_r = 180 - 135°$

$= 45°$

$\tan 135° = -1$

f) $\sin 150^0$

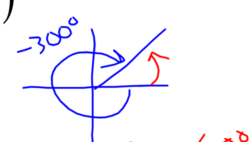


$\theta_r = 180 - 150°$

$= 30°$

$\sin 150° = \frac{1}{2}$

g) $\tan(-300^0)$



$\theta_r = 60°$

$\tan(-300°) = \frac{\sqrt{3}}{1}$

$= \sqrt{3}$

.

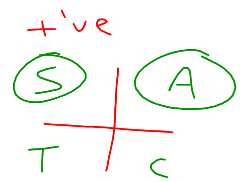
Ex. 2 Determine <u>all</u> possible values for $0 < \theta < 360^0$.

Pull

<u>Process</u>
1. Determine the quadrants.
2. Draw a diagram with terminal arms.
3. Determine the related angle.
4. Find the principal angles.

a) $\sin\theta = \frac{1}{\sqrt{2}}$

Sign is +'ve $\therefore$



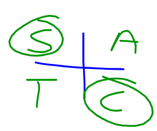
$\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$
$= 45^\circ$

<u>Q1</u>
$\theta = 45^\circ$

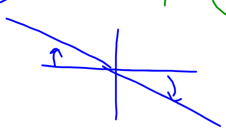
<u>Q2</u>
$\theta = 180 - 45^\circ$
$= 135^\circ$

$\therefore \theta = 45^\circ, 135^\circ$

b) $\tan\theta = -\frac{1}{\sqrt{3}}$



$\theta_r = 30^\circ$

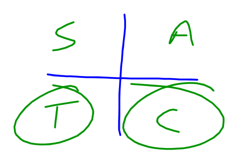


<u>Q2</u>
$\theta = 180 - 30^\circ$
$= 150^\circ$

<u>Q4</u>
$\theta = 360 - 30$
$= 330^\circ$

$\therefore \theta = 150^\circ, 330^\circ$

c) $\sin\theta = \frac{-\sqrt{3}}{2}$



$\theta_r = 60^\circ$

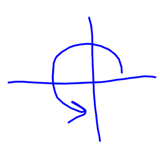
<u>Q3</u>
$\theta = 180 + 60$
$= 240$

<u>Q4</u>
$\theta = 360 - 60$
$= 300^\circ$

$\therefore \theta = 240, 300^\circ$

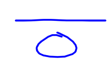
d) $\sin\theta = -1$

$\theta = 270^\circ$



e) $\tan\theta = undefined$

$\theta = 90^\circ, 270^\circ$



Ex 3: Evaluate the following using exact values.

$$\sin 30^0 \cos^2\left(225^0\right) - \tan\left(-60^0\right)$$

$$= \underbrace{\sin 30^\circ}\underbrace{\left[\cos 225^\circ\right]^2} - \underbrace{\tan(-60^\circ)}$$

$$= \frac{1}{2} \left(-\frac{\sqrt{2}}{2}\right)^2 - \left(-\frac{\sqrt{3}}{1}\right)$$

$$= \frac{1}{2}\left(\frac{2}{4}\right) + \sqrt{3}$$

$$= \frac{1}{4} + \sqrt{3}\left(\frac{4}{4}\right)$$

$$= \frac{1}{4} + \frac{4\sqrt{3}}{4}$$

$$= \frac{1+4\sqrt{3}}{4}$$

p.228 #3b, 6, 17
p.237 #6ace, 7, 9, 10, 16