## Lesson 4.3A Reciprocal Trigonometric Ratios

In every right triangle, there are three primary trig ratios and three reciprocal trig ratios.

The reciprocal trig ratios are:

$$
\begin{aligned}
& \text { cosecant } \\
& \text { reciprocal of stan } \\
& \csc \theta=\frac{1}{\sin \theta} \\
& =\frac{1}{\left(\frac{y}{r}\right)} \\
& =\frac{r}{y} \\
& \text { secant } \\
& \text { reciprocal of cos } \\
& \sec \theta=\frac{1}{\cos \theta} \\
& =\frac{1}{\left(\frac{x}{r}\right)} \\
& =\frac{r}{x} \\
& \text { reciprocal of tan } \\
& \cot \theta=\frac{1}{\tan \theta} \\
& =\frac{1}{\left(\frac{y}{x}\right)} \\
& =\frac{x}{y}
\end{aligned}
$$

Ex. 1 Find the exact values of the following.
a) $\sec 45^{0}$
$\cos 45^{\circ}=\frac{1}{\sqrt{2}}$
$\sec 45^{\circ}=\frac{\sqrt{2}}{1}$
b) $\cot 120^{\circ} \mathrm{s}$

| $0^{0} S$ | $A$ |
| :---: | :---: |
|  | $120^{\circ}$ |
| $T$ | $C$ |
| 0 | $=180$ |

c) $\csc 210^{\circ}$

$$
=\sqrt{2}
$$

$\tan 120^{\circ}=-\frac{\sqrt{3}}{1}$

$$
\begin{aligned}
\theta_{r} & =180-120 \\
& =60^{\circ}
\end{aligned}
$$

$$
\sin 210^{\circ}=-\frac{1}{2}
$$

$\cot 120^{\circ}=-\frac{1}{\sqrt{3}}$

$$
=60^{\circ} \quad \begin{aligned}
\csc 210^{\circ} & =-\frac{2}{1} \\
& =-2
\end{aligned}
$$

d) $\csc 180^{\circ}$
$\sin 180^{\circ}=\frac{0}{1}$
$\csc 180^{\circ}=\frac{1}{0}$
UNDEFINED!

$$
\begin{aligned}
& \text { e) } \cot 675^{\circ} \\
& \tan 675^{\circ}-360^{\circ} \\
& =\tan 315^{\circ} \\
& \tan 315^{\circ}=-1 \\
& \therefore \cot 675^{\circ}=-1
\end{aligned}
$$


$\theta_{r}=360-3 / 5$

$$
=45^{\circ}
$$



Ex. 2 Find the exact yalues of $\theta$ for $0 \leq \theta \leq 360^{\circ}$.
a) $\sec \theta=-\frac{2}{\sqrt{3}}$
b) $\csc \theta=-\sqrt{2}$
$\sin \theta=-\frac{1}{\sqrt{2}}$
$\theta_{r}=45^{\circ}$
$\theta_{r}=30^{\circ}$


$$
\begin{array}{c|c}
S & A \\
\hline T & C
\end{array}
$$

$$
\begin{aligned}
& Q 2 \\
& \theta=180-30 \\
& =150^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
& Q 3 \\
& \theta=180+30 \\
& =210^{\circ}
\end{aligned}
$$

$$
\begin{array}{cc}
\frac{Q 3}{\theta=180+45} & \frac{Q 4}{\theta=360-45} \\
=225^{\circ} & =315^{\circ}
\end{array}
$$

$$
\therefore \theta=150^{\circ}, 210^{\circ}
$$

$$
\theta=225^{\circ}, 315^{\circ}
$$

c) $\cot \theta=0$
$\tan \theta=\frac{1}{0}$
UNDEFINED

$(0,-1)$
$\theta=90^{\circ} .270^{\circ}$

Ex. 3 Evaluate the following to four decimal places. Use calculators!
a) $\csc 67^{0}$
b) $\cot 342^{\circ}$
c) $\sec 143^{\circ}$
$=\frac{1}{\sin 67^{\circ}}$
$=\frac{1}{\tan 342^{\circ}}$
$=\frac{1}{\cos 143^{\circ}}$
$=1.0864$
$=-3.0777$
$=-1.2521$

$$
1 / x
$$

d) $\sec \left(-104^{\circ}\right)$
$=\frac{1}{\cos \left(-104^{\circ}\right)}$
$=-4.1336$

Ex. 4 Determine the value of $\theta$ for $0^{0} \leq \theta \leq 360^{\circ}$. Use your calculators. Round to the nearest degree.
a) $\csc \theta=-1.7842$
$\sin \theta=-\frac{1}{1.7842}$

$$
\theta=\sin ^{-1}\left(-\frac{1}{1.7842}\right)
$$

$$
=-34^{\circ}
$$

$$
\begin{aligned}
& \text { Q3 } \\
& \begin{aligned}
\theta & =180+34^{\circ} \\
& =214^{\circ}
\end{aligned}
\end{aligned}
$$

$$
\theta_{r}=34^{\circ}
$$

$$
\begin{aligned}
\frac{Q 4}{\theta} & =360-34^{\circ} \\
& =326^{\circ}
\end{aligned}
$$

$$
\therefore \theta=214^{\circ}, 326^{\circ}
$$

b) $\quad$ cot $=-0.4663$

$$
\tan \theta=-\frac{1}{0.4663}
$$

$$
\theta=-65^{\circ}
$$



$$
\begin{aligned}
\theta & \frac{Q 2}{}=180-65 & \begin{aligned}
& Q 4 \\
&=115^{\circ}
\end{aligned} & \begin{aligned}
\theta & =360-65 \\
& =295^{\circ}
\end{aligned}
\end{aligned}
$$

$$
\theta=115^{\circ}, 295^{\circ}
$$

## Homework

p. 246 \#1af,2,4,5acgh,6,7,8ace,10,13,16

