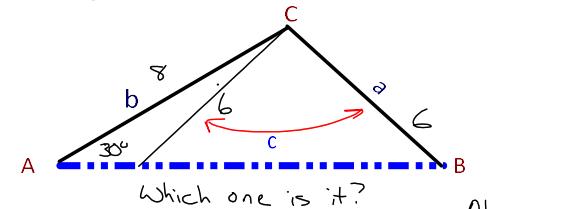
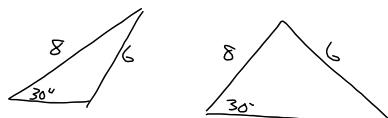


Lesson 4.4B: Sine Law - AMBIGUOUS Case

Draw triangle ΔABC , $a = 6 \text{ cm}$, $b = 8 \text{ cm}$, $A = 30^\circ$.

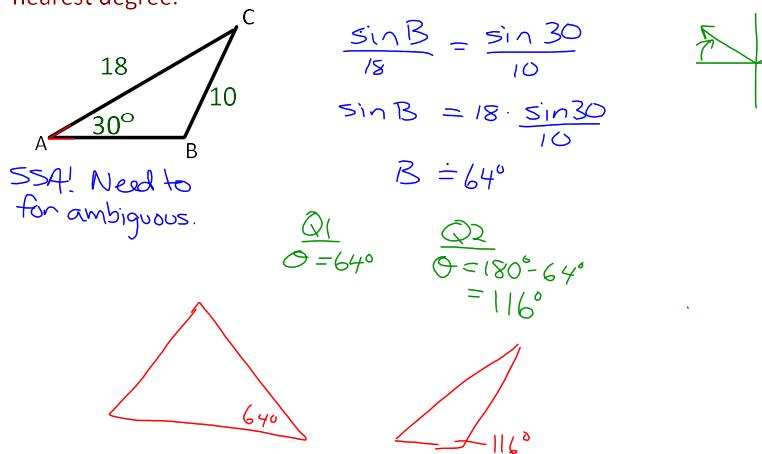


Always need to account for both.

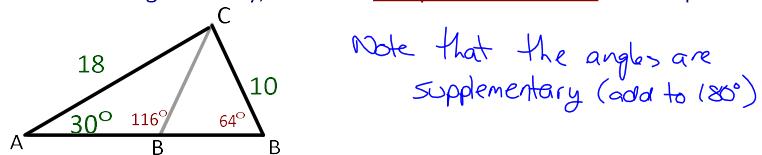


- When two sides and the non-included angle of a triangle are given, the triangle may not be unique. (SSA)
- You will have to determine if there is **no solution**, **one solution** or **two possible solutions**.

Ex. 1: Given that ΔABC has $\angle A = 30^\circ$, $a = 10$, and $b = 18$, find the value of $\angle B$ to the nearest degree.



As we see algebraically, there are two possible answers to this question.



Therefore, it is very important to always **consider** both solutions (Q1 & Q2) when using Sine Law to solve a triangle given SSA.

$$\begin{aligned} \angle C &= 180 - 30 - 64 \\ &= 86^\circ \end{aligned}$$

(VALID)

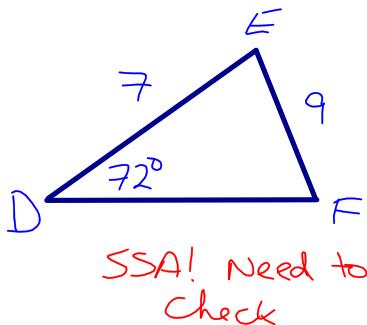
$$\begin{aligned} \angle C &= 180 - 30 - 116 \\ &= 34^\circ \end{aligned}$$

(VALID)

\therefore There are two solutions!

Ex. 2: Determine the measures of all angles in the given triangles.

a) In $\triangle DEF$, $\angle D = 72^\circ$, $d = 9 \text{ cm}$, $f = 7 \text{ cm}$.



$$\frac{\sin F}{7} = \frac{\sin 72^\circ}{9}$$

$$\sin F = 7 \cdot \frac{\sin 72^\circ}{9}$$

$$F \approx 47.7^\circ$$



$$\begin{aligned} Q1: \quad & \frac{Q1}{\theta} = 47.7 \\ Q2: \quad & \theta = 180^\circ - 47.7^\circ \\ & = 132.3^\circ \end{aligned}$$

Triangle 1

$$\begin{aligned} \angle E &= 180^\circ - 72^\circ - 47.7^\circ \\ &= 60.3^\circ \end{aligned}$$

VALID

\therefore Only 1 solution

$$\angle E = 60.3^\circ$$

$$\angle F = 47.7^\circ$$

$$\angle D = 72^\circ$$

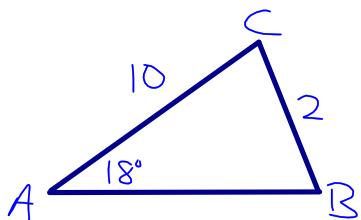
Triangle 2

$$\begin{aligned} \angle E &= 180^\circ - 72^\circ - 132.3^\circ \\ &= -24.3^\circ \end{aligned}$$

INVALID

Pull

b) In $\triangle ABC$, $\angle A = 18^\circ$, $a = 2 \text{ cm}$, $b = 10 \text{ cm}$.

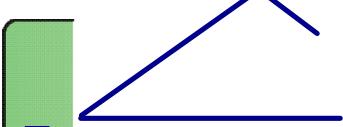


$$\frac{\sin B}{10} = \frac{\sin 18^\circ}{2}$$

$$\sin B = 10 \cdot \frac{\sin 18^\circ}{2}$$

$B = \text{ERROR}$

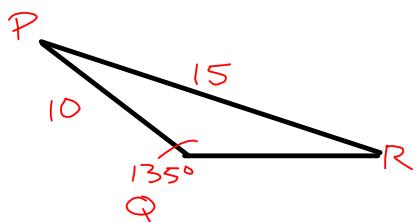
\therefore NO SOLUTIONS



Pull

This example shows the case of no solution.
The triangle cannot be constructed as side "a" is too short.

c) In ΔPQR , $\angle Q = 135^\circ$, $q = 15 \text{ cm}$, $r = 10 \text{ cm}$.



$$\frac{\sin R}{10} = \frac{\sin 135^\circ}{15}$$

$$R \approx 28.1^\circ$$

SSA! Need to check

$$\theta_1 = 28.1^\circ$$

$$\begin{aligned}\theta_2 &= 180 - 28.1 \\ &= 151.9^\circ\end{aligned}$$

Both valid?
Triangle 1

$$\begin{aligned}\angle P &= 180 - 135 - 28.1 \\ &= 16.9^\circ\end{aligned}$$

(VALID)

Triangle 2

$$\begin{aligned}\angle P &= 180 - 135 - 151.9 \\ &= 19.1^\circ\end{aligned}$$

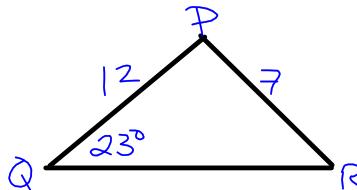
(INVALID)

Pull

\therefore Only 1 solution

$$\begin{aligned}\angle P &= 16.9^\circ \\ \angle R &= 28.1^\circ \\ \angle Q &= 135^\circ\end{aligned}$$

d) In ΔPQR , $\angle Q = 23^\circ$, $q = 7 \text{ cm}$, $r = 12 \text{ cm}$.



$$\frac{\sin R}{12} = \frac{\sin 23}{7}$$

$$R \approx 42.1^\circ$$

ASSA \rightarrow Need to check

$$\theta_1 = 42.1^\circ$$

$$\begin{aligned}\theta_2 &= 180 - 42.1 \\ &= 137.9^\circ\end{aligned}$$

Triangle 1

$$\begin{aligned}\angle Q &= 23^\circ \\ \angle R &= 42.1^\circ \\ \angle P &= 180 - 23 - 42.1 \\ &= 114.9\end{aligned}$$

(VALID)

Triangle 2

$$\begin{aligned}\angle Q &= 23^\circ \\ \angle R &= 137.9^\circ \\ \angle P &= 180 - 23 - 137.9 \\ &= 19.1^\circ\end{aligned}$$

(VALID)

Pull

\therefore Two solutions!

$$\angle Q = 23^\circ$$

$$\angle R = 42.1^\circ$$

$$\angle P = 114.9^\circ$$

$$\angle Q = 23^\circ$$

$$\angle R = 137.9^\circ$$

$$\angle P = 19.1^\circ$$

Homework
pg 254 #6a, 14bcd, 16, 21abc

am•bi•gu•ous

doubtful, uncertain,
unclear in meaning