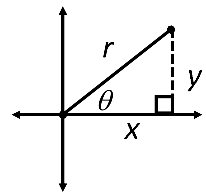
Lesson 4.6 A: Trig Identities (Day 1)

Recall the following definitions:



$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$ $x^2 + y^2 = r^2$

An identity is an equation that is always true, regardless of the value of the variable.

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

1) The Quotient Identities

$$\frac{\sin\theta}{\cos\theta} =$$

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$$\frac{\sin\theta}{\cos\theta} = \frac{y}{r} \div \frac{x}{r}$$

$$= \frac{y}{\cancel{r}} \times \frac{\cancel{r}}{x}$$

$$= \frac{y}{x}$$

$$= \tan\theta$$

$$\boxed{\frac{\sin\theta}{\cos\theta} = \tan\theta} \qquad \boxed{\frac{\cos\theta}{\sin\theta} = \cot\theta}$$

Note:

$$\boxed{\frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta} \qquad \boxed{\frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta}$$

2) The Pythagorean Identities

$$\sin^2\theta + \cos^2\theta =$$

$$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$

$$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$$

$$= \frac{y^2 + x^2}{r^2}$$

$$= \frac{r^2}{r^2}$$

$$= 1$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

This identity is often rearranged to give:

$$\boxed{\sin^2\theta = 1 - \cos^2\theta}$$

$$\boxed{\cos^2\theta = 1 - \sin^2\theta}$$

3) The Reciprocal Identities

$\sin\theta = \dfrac{1}{\csc\theta}$	$\cos\theta = \dfrac{1}{\sec\theta}$	$\tan\theta = \dfrac{1}{\cot\theta}$
$\csc\theta = \dfrac{1}{\sin\theta}$	$\sec\theta = \dfrac{1}{\cos\theta}$	$\cot\theta = \dfrac{1}{\tan\theta}$

These identities can also be squared.

STEPS TO PROVING IDENTITIES

1. Separate LS from RS.
2. Write both sides in terms of sin x and cos x.
3. To make LS = RS , try :
   - Factoring.
   - Simplifying.
   - Substitute any of the identities we just learned.
   - In some situations, multiply by the conjugate.

Examples: Prove each identity.

a) $\cos x \tan x = \sin x$

L.S.	R.S.
$\cos x \tan x$	$\sin x$
$= \dfrac{\cancel{\cos x}}{1} \cdot \dfrac{\sin x}{\cancel{\cos x}}$	
$= \sin x$	

$\therefore LS = RS$

$\therefore$ Q.E.D.

PULL

"Quod Erat Demonstrandum"
Which is what had to be proven.

b) $\dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x} = \dfrac{2}{\cos^2 x}$

LS	RS
$\dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x}$	$\dfrac{2}{\cos^2 x}$
$= \dfrac{1-\sin x}{1-\sin x} \cdot \dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x} \cdot \dfrac{1+\sin x}{1+\sin x}$	$= \dfrac{2}{1-\sin^2 x}$
$= \dfrac{1-\sin x + 1+\sin x}{(1+\sin x)(1-\sin x)}$	
$= \dfrac{2}{1 \cancel{-\sin x} \cancel{+\sin x} - \sin^2 x}$	
$= \dfrac{2}{1-\sin^2 x}$	

$\therefore LS = RS$

$\therefore$ QED

c) $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$

LS	RS
$\tan^2 x - \sin^2 x$	$\sin^2 x \tan^2 x$
$= \dfrac{\sin^2 x}{\cos^2 x} - \sin^2 x \cdot \dfrac{\cos^2 x}{\cos^2 x}$	$= \sin^2 x \cdot \dfrac{\sin^2 x}{\cos^2 x}$
$= \dfrac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$	$= \dfrac{\sin^4 x}{\cos^2 x}$
$= \dfrac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$	
$= \dfrac{\sin^2 x (\sin^2 x)}{\cos^2 x}$	
$= \dfrac{\sin^4 x}{\cos^2 x}$	

$\therefore LS = RS$

$\therefore$ QED

d) $\dfrac{\cos x - \sin x - \cos^3 x}{\cos x} = \sin^2 x - \tan x$

LS	RS
$\dfrac{\cos x - \sin x - \cos^3 x}{\cos x}$	$= \sin^2 x - \tan x$
	$= \sin^2 x - \dfrac{\sin x}{\cos x}$
	$= \dfrac{\cos x \sin^2 x}{\cos x} - \dfrac{\sin x}{\cos x}$
	$= \dfrac{\cos x \sin^2 x - \sin x}{\cos x}$
	$= \dfrac{\cos x (1 - \cos^2 x) - \sin x}{\cos x}$
	$= \dfrac{\cos x - \cos^3 x - \sin x}{\cos x}$

$\therefore LS = RS$

$\therefore$ QED