

Lesson 4.6B - Trig Identities (Day 2)

Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\csc^2 \theta = \frac{1}{\sin^2 \theta}$$

$$\sec^2 \theta = \frac{1}{\cos^2 \theta}$$

$$\cot^2 \theta = \frac{1}{\tan^2 \theta}$$

Quotient Identities

$$\star \tan \theta = \frac{\sin \theta}{\cos \theta}$$

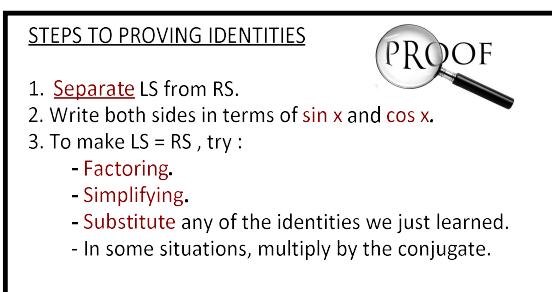
$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\cot^2 \theta = \frac{\cos^2 \theta}{\sin^2 \theta}$$

Pythagorean Identities

$$\star \sin^2 \theta + \cos^2 \theta = 1$$

StrategiesFactoring

$$1 - \cos^2 \theta \quad \text{Diff. of squares}$$

$$= (1 + \cos \theta)(1 - \cos \theta)$$

$$\begin{aligned} & \sin x - \sin^2 x \\ &= \sin x(1 - \sin x) \end{aligned}$$

$$\begin{aligned} \sin^2 \theta - 2 \sin \theta + 1 & \quad M \quad 1 \\ & \quad A \quad -2 \\ & \quad N \quad -1, -1 \\ &= (\sin \theta - 1)^2 \end{aligned}$$

$$\begin{aligned} \sin^2 \theta - \cos^2 \theta & \quad \text{Diff. of squares!} \\ & \quad x^2 - y^2 \\ &= (\sin \theta - \cos \theta)(\sin \theta + \cos \theta) \end{aligned}$$

$$\begin{aligned} \cos^2 \theta - 7 \cos \theta + 10 & \quad M \quad 10 \\ & \quad A \quad -7 \\ & \quad N \quad -2, -5 \\ &= (\cos \theta - 2)(\cos \theta - 5) \end{aligned}$$

$$\begin{aligned} 6 \sin^2 \theta - \sin \theta - 1 & \quad M \quad -6 \\ & \quad A \quad -1 \\ & \quad N \quad -\frac{6}{3}, \frac{6}{2} \\ & \quad \frac{2}{-1} \quad \frac{3}{1} \\ &= (2 \sin \theta - 1)(3 \sin \theta + 1) \end{aligned}$$

Multiplying by the conjugate

$$\begin{aligned} & \frac{1}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} \\ &= \frac{1 + \cos x}{1 + \cos x - \cos x - \cos^2 x} \\ &= \frac{1 + \cos x}{1 - \cos^2 x} \end{aligned}$$

Examples - Prove the following identities.

a) $\frac{1}{\cot x} = \sin x \sec x$

LS	RS
$\frac{1}{\cot x}$ $= \tan x$ $= \frac{\sin x}{\cos x}$	$\sin x \sec x$ $= \sin x \frac{1}{\cos x}$ $= \frac{\sin x}{\cos x}$
	$\therefore LS = RS$ $\therefore QED!$

b) $\frac{1 + \cot x}{\csc x} = \sin x + \cos x$

LS	RS
$\frac{1 + \cot x}{\csc x}$ $= (1 + \frac{\cos x}{\sin x}) \div (\frac{1}{\sin x})$ $= (1 + \frac{\cos x}{\sin x}) \times (\frac{\sin x}{1})$ $= \sin x + \frac{\cos x \cancel{\sin x}}{\cancel{\sin x}}$ $= \sin x + \cos x$	$\sin x + \cos x$
	$\therefore LS = RS$ $\therefore QED$

$$\text{c) } \frac{\cos\theta - 1}{1 - \sec\theta} = \frac{\cos\theta + 1}{1 + \sec\theta}$$

LS	RS
$\frac{\cos\theta - 1}{1 - \sec\theta}$ $= (\cos\theta - 1) \div \left(1 - \frac{1}{\cos\theta}\right)$ $= (\cos\theta - 1) \div \left(\frac{\cos\theta - 1}{\cos\theta}\right)$ $= \frac{\cancel{(\cos\theta - 1)}}{1} \times \frac{\cos\theta}{\cancel{(\cos\theta - 1)}}$ $= \cos\theta$	$\frac{\cos\theta + 1}{1 + \sec\theta}$ $= (\cos\theta + 1) \div \left(1 + \frac{1}{\cos\theta}\right)$ $= (\cos\theta + 1) \div \left(\frac{\cos\theta + 1}{\cos\theta}\right)$ $= \frac{\cancel{(\cos\theta + 1)}}{1} \times \frac{\cos\theta}{\cancel{(\cos\theta + 1)}}$ $= \cos\theta$
$\therefore LS = RS$ $\therefore QED$	

d) $\tan \alpha \sin \alpha + \cos \alpha = \sec \alpha$

LS $\begin{aligned} & \tan \alpha \sin \alpha + \cos \alpha \\ &= \frac{\sin \alpha}{\cos \alpha} \cdot \sin \alpha + \cos \alpha \\ &= \frac{\sin^2 \alpha}{\cos \alpha} + \cos \alpha \quad \frac{\cos \alpha}{\cos \alpha} \\ &= \frac{\sin^2 \alpha}{\cos \alpha} + \frac{\cos^2 \alpha}{\cos \alpha} \\ &= \frac{\sin^2 \alpha + \cos^2 \alpha}{\cos \alpha} \\ &= \frac{1}{\cos \alpha} \end{aligned}$	RS $\begin{aligned} & \sec \alpha \\ &= \frac{1}{\cos \alpha} \\ \\ &\therefore \text{LS} = \text{RS} \\ &\therefore \text{QED} \end{aligned}$
---	---

e) $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$

LS $\begin{aligned} & \sin^4 x - \cos^4 x \\ &= (\sin^2 x - \cos^2 x)(\underbrace{\sin^2 x + \cos^2 x}_1) \\ &= \sin^2 x - \cos^2 x \end{aligned}$	RS $\begin{aligned} & \sin^2 x - \cos^2 x \\ \\ &\therefore \text{LS} = \text{RS} \\ &\therefore \text{QED} \end{aligned}$
---	---

f) $\sin x - \sin x \cos^2 x = \sin^3 x$

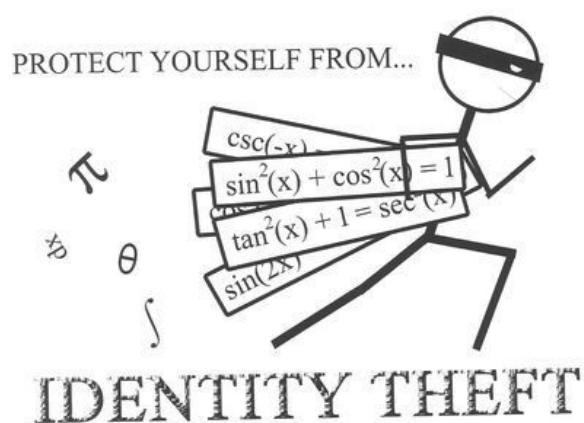
LS	RS
$\begin{aligned} &\sin x - \sin x \cos^2 x \\ &= \sin x (1 - \cos^2 x) \\ &= \sin x (\sin^2 x) \\ &= \sin^3 x \end{aligned}$	$\begin{aligned} &\sin^3 x \\ &\therefore LS = RS \\ &\therefore QED \end{aligned}$

g) $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

LS	RS
$\begin{aligned} &(\sin x - \cos x)^2 + (\sin x + \cos x)^2 \\ &= \sin^2 x - 2\sin x \cos x + \cos^2 x + \sin^2 x + 2\sin x \cos x + \cos^2 x \\ &= \sin^2 x + \cos^2 x + \sin^2 x + \cos^2 x \\ &= 1 + 1 \\ &= 2 \end{aligned}$	$\begin{aligned} &(a-b)^2 \\ &= a^2 - 2ab + b^2 \\ &= 2 \end{aligned}$

h) $\frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \frac{3 + \sin x}{1 - \sin x}$

LS	RS
$\begin{aligned} &\frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} \\ &= \frac{(\sin x + 3)(\sin x + 1)}{\cos^2 x} \\ &= \frac{(\sin x + 3)(\sin x + 1)}{1 - \sin^2 x} \end{aligned}$	$\begin{aligned} &\frac{3 + \sin x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} \\ &= \frac{(3 + \sin x)(1 + \sin x)}{1 - \sin x + \sin x - \sin^2 x} \\ &= \frac{(3 + \sin x)(1 + \sin x)}{1 - \sin^2 x} \\ &\therefore LS = RS \\ &\therefore QED \end{aligned}$



created by: David Ritzenthaler

3.14.2013