# Combinations Worksheet Solutions

**1.** We are choosing 3 distinct students out of 10 without regard to order:

$$\binom{10}{3} = 120.$$

**2.** First, choose exactly 2 reds from 5:

$$\binom{5}{2} = 10.$$

You need 3 more marbles chosen from the non-red marbles (4 blue + 3 green = 7 total):

$$\binom{7}{3} = 35.$$

Total number of ways:

$$10 \times 35 = \boxed{350}.$$

**3.** Possible distributions (F,B,P) that sum to 4 with at least one from each:
(1F,1B,2P), (1F,2B,1P), (2F,1B,1P)

Compute each:
(1F,1B,2P):

$$\binom{3}{1}\binom{3}{1}\binom{2}{2} = 3 \times 3 \times 1 = 9.$$

(1F,2B,1P):

$$\binom{3}{1}\binom{3}{2}\binom{2}{1} = 3 \times 3 \times 2 = 18.$$

(2F,1B,1P):

$$\binom{3}{2}\binom{3}{1}\binom{2}{1} = 3 \times 3 \times 2 = 18.$$

Total:

$$9 + 18 + 18 = \boxed{45}.$$

**4.** First choose the 5-person team:

$$\binom{12}{5}.$$

Then choose the 4-person team from the remaining 7:

$$\binom{7}{4}.$$

The leftover 3 are automatically not chosen. Thus:

$$\binom{12}{5}\binom{7}{4}.$$

Numerically: $\binom{12}{5} = 792$ and $\binom{7}{4} = 35$.
Total:

$$792 \times 35 = \boxed{27720}.$$

**5.** Choose at least one piece of fruit from: 4 apples, 5 bananas, 2 cantaloupes, 3 pears.
Number of ways (including the case of "zero"):

$$(4+1)(5+1)(2+1)(3+1) = 5 \times 6 \times 3 \times 4 = 360.$$

Subtract the empty selection (the case of all being zero):

$$360 - 1 = \boxed{359}.$$

**6.** There are 25 students and 6 first-class seats (FC) and 19 economy seats (E). Alex (A) and Heather (H) must sit in FC. Rachel (R), Jenna (J), and David (D) must sit in E. The other 20 students have no restrictions.

$$\underbrace{A, H}_{\text{FC}} \quad | \quad \underbrace{R, J, D}_{\text{E}} \quad | \quad \text{20 unrestricted}$$

After placing A and H in FC, we have:

$$6 - 2 = 4 \text{ FC seats left.}$$

Now we must select which 4 of the 20 unrestricted students will join A and H in First Class. Once these 4 are chosen, the remaining 16 must go to E. The number of ways to choose these 4 students is:

$$\binom{20}{4} = \boxed{4845}.$$

**7.** 3 teachers each choose and rank 3 out of 15 students. Lists share exactly one common name, placed in a different rank by each teacher.

Steps:

$$\text{Choose common student: 15.}$$

$$\text{Permute that student in 3 ranks among 3 teachers: } 3! = 6.$$

Remaining 14 students to form three pairs (2 for each teacher's list besides the common student):

$$\binom{14}{2}\binom{12}{2}\binom{10}{2} = 91 \times 66 \times 45 = 270{,}270.$$

Each teacher arranges their chosen 3 in order, but the common student's position is fixed. The other 2 can be arranged in:

$$2! \text{ ways per teacher, total } 2!^3 = 8.$$

Multiply all:

$$15 \times 6 \times 270{,}270 \times 8 = \boxed{194{,}594{,}400}.$$

**8.** We consider all the possible arrangements, and then add them up. For example: for 4 courses in group A, that means we can only have 1 in group B and 1 in group C. There are six arrangements like this in total here:

(2A,1B,3C), (2A,2B,2C), (2A,3B,1C), (3A,1B,2C), (3A,2B,1C), (4A,1B,1C)

Calculate each:

(2A,1B,3C):

$$\binom{4}{2}\binom{3}{1}\binom{3}{3} = 6 \times 3 \times 1 = 18.$$

(2A,2B,2C):

$$\binom{4}{2}\binom{3}{2}\binom{3}{2} = 6 \times 3 \times 3 = 54.$$

(2A,3B,1C):

$$\binom{4}{2}\binom{3}{3}\binom{3}{1} = 6 \times 1 \times 3 = 18.$$

(3A,1B,2C):

$$\binom{4}{3}\binom{3}{1}\binom{3}{2} = 4 \times 3 \times 3 = 36.$$

(3A,2B,1C):

$$\binom{4}{3}\binom{3}{2}\binom{3}{1} = 4 \times 3 \times 3 = 36.$$

(4A,1B,1C):

$$\binom{4}{4}\binom{3}{1}\binom{3}{1} = 1 \times 3 \times 3 = 9.$$

Sum them up:

$$18 + 54 + 18 + 36 + 36 + 9 = \boxed{171}.$$