MDM4U Chapter 4: Introduction to Probability Assignment Solutions

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Question 1

A road has two stop lights at two consecutive intersections. The probability of getting a green light at the first intersection is 0.6, and the probability of getting a green light at the second intersection, given that you got a green light at the first intersection, is 0.8.

Solution

The probability of getting a green light at both intersections is given by:

$$\begin{aligned} P(\text{Green at both}) &= P(\text{Green at first}) \times P(\text{Green at second} \mid \text{Green at first}) \\ &= 0.6 \times 0.8 = 0.48 \end{aligned}$$

Question 2

A bag contains three red marbles and five white marbles. What is the probability of drawing two red marbles at random if the first marble drawn is not replaced?

Solution

The probability of drawing two red marbles without replacement is:

$$\begin{aligned} P(\text{Two Reds}) &= P(\text{First red}) \times P(\text{Second red} \mid \text{First red}) \\ &= \frac{3}{8} \times \frac{2}{7} \\ &= \frac{6}{56} \\ &= \frac{3}{28} \end{aligned}$$

Question 3

A deck with two joker cards (one red, one black) has 54 cards. A single card is drawn and then a second card is drawn without replacement.
(This is terrible math form. Equal signs should be aligned like question 2)

a) Probability of drawing one joker on the first draw and an ace on the second draw

$$P(\text{Joker first, Ace second}) = \frac{2}{54} \times \frac{4}{53} = \frac{8}{2862} = \frac{4}{1431}$$

b) Probability of drawing a numbered card on the first draw and the red joker on the second draw

$$P(\text{Numbered first, Red joker second}) = \frac{36}{54} \times \frac{1}{53} = \frac{36}{2862} = \frac{2}{159}$$

c) Probability of drawing a queen on both draws

$$P(\text{Queen both draws}) = \frac{4}{54} \times \frac{3}{53} = \frac{12}{2862} = \frac{6}{1431}$$

d) Probability of drawing any black card on both draws

$$P(\text{Black both draws}) = \frac{27}{54} \times \frac{26}{53} = \frac{702}{2862} = \frac{13}{53}$$

Question 4

Roger Federer made 60% of his first serves in 2011. When he made his first serve, he won 75% of the points. When he missed, he won only 50% of the points.

b) Probability that Federer makes the first serve and wins the point

$$\begin{aligned} P(\text{Make and win}) &= 0.6 \times 0.75 \\ &= 0.45 \end{aligned}$$

c) Probability that he loses the point

$$\begin{aligned} P(\text{Lose}) &= P(\text{Make and lose}) + P(\text{Miss and lose}) \\ &= (0.6 \times 0.25) + (0.4 \times 0.5) \\ &= 0.15 + 0.2 \\ &= 0.35 \end{aligned}$$

Question 5

Eight students are to line up for a photograph.

a) Total arrangements

$$8! = 40,320$$

b) Arrangements if Jill is first

$$7! = 5,040$$

c) Arrangements if Jill is first and Meera is last

$$6! = 720$$

Question 6

Guessing the correct order of the first 9 elements in the periodic table.

Solution

$$9! = 362,880$$

$$P(\text{Correct guess}) = \frac{1}{362,880}$$

Question 7

Solve for n:

$$\frac{(n-1)!}{(n-3)!} = 20$$

Expanding, we get:

$$\begin{aligned}
\frac{(n-1)!}{(n-3)!} &= 20 \\
\frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} &= 20 \\
(n-1)(n-2) &= 20 \\
n^2 - 3n + 2 &= 20 \\
n^2 - 3n - 18 &= 0 \\
(n-6)(n+3) &= 0
\end{aligned}$$

$$\therefore n = 6 \text{ or } \cancel{n = -3}$$

(n cannot be negative, because negative factorials are invalid)

Question 8: Arrangements from the Letters of "ORANGES"

Letters: O, R, A, N, G, E, S.
Vowels = O, A, E (3 distinct vowels)
Consonants = R, N, G, S (4 distinct consonants)

(a) **Two vowels and two consonants (4-letter word):**

$$\binom{3}{2} = 3 \quad \text{(choosing 2 vowels from 3)}$$

$$\binom{4}{2} = 6 \quad \text{(choosing 2 consonants from 4)}$$

$$4! = 24 \quad \text{(arranging 4 distinct letters)}$$

$$\text{Total} = 3 \times 6 \times 24 = 432.$$

(b) **Two vowels and three consonants (5-letter word):**

$$\binom{3}{2} = 3 \quad \text{(choosing 2 vowels from 3)}$$

$$\binom{4}{3} = 4 \quad \text{(choosing 3 consonants from 4)}$$

$$5! = 120 \quad \text{(arranging 5 distinct letters)}$$

$$\text{Total} = 3 \times 4 \times 120 = 1440.$$

(c) **Three letters from "ORANG" and one letter from "ES" (4-letter word):**

"ORANG" has 5 distinct letters. "ES" has 2 distinct letters.

$$\binom{5}{3} = 10 \quad \text{(choosing 3 from ORANG)}$$

$$\binom{2}{1} = 2 \quad \text{(choosing 1 from ES)}$$

$$4! = 24 \quad \text{(arranging the 4 chosen distinct letters)}$$

$$\text{Total} = 10 \times 2 \times 24 = 480.$$

(d) **From the sentence "KITY LUVZ ORANGES" — form a 7-letter word using 2 letters from "KITY", 2 letters from "LUVZ", and 3 letters from "ORANGES":**

"KITY" has 4 distinct letters: {K, I, T, Y}

$$\binom{4}{2} = 6$$

"LUVZ" has 4 distinct letters: {L, U, V, Z}

$$\binom{4}{2} = 6$$

"ORANGES" has 7 distinct letters: {O, R, A, N, G, E, S}

$$\binom{7}{3} = 35$$

$$7! = 5040 \quad \text{(arranging the 7 chosen letters)}$$

$$\text{Total} = 6 \times 6 \times 35 \times 5040 = 6{,}350{,}400.$$

Question 9

Five men and four women form a committee.

a) No restrictions

$$\binom{9}{5} = 126$$

b) Exactly three women

$$\binom{4}{3} \times \binom{5}{2} = 40$$

c) Exactly four men

$$\binom{5}{4} \times \binom{4}{1} = 20$$

d) No women

$$\binom{5}{5} = 1$$

e) At least two men

$$126 - 5 = 121$$

f) At least three women

$$40 + 5 = 45$$

Question 10

A professor grades 5 of 12 homework problems.

a) Total groups of 5

$$\binom{12}{5} = 792$$

b) Probability that Jerry's 5 problems are selected

$$\frac{1}{792}$$

c) Silvia completed 7 problems; probability of a match

$$\binom{7}{5} = 21 \Rightarrow \frac{21}{792} = \frac{7}{264}$$

Question 11

A bag contains 10 gum balls, 7 candy bars, and 3 toffees. Two candies are drawn without replacement.

a) Probability of keeping a gum ball

$$\frac{10}{20} \times \frac{9}{19} = \frac{9}{38}$$

b) Probability of keeping any candy

$$\frac{9}{38} + \frac{21}{190} + \frac{3}{190} = \frac{69}{190}$$

c) Probability of not keeping any candy

$$1 - \frac{69}{190} = \frac{121}{190}$$

Quick Summary of Answers

1. 0.48
2. $\frac{3}{28}$
3. (a) $\frac{4}{1431}(0.3\%)$, (b) $\frac{2}{159}(1.3\%)$, (c) $\frac{2}{477}(0.4\%)$, (d) $\frac{13}{53}(24.5\%)$
4. (b) 0.45, (c) 0.35
5. (a) $8! = 40{,}320$, (b) $7! = 5{,}040$, (c) $6! = 720$
6. $\frac{1}{9!} = \frac{1}{362880}$
7. $n = 6$
8. (a) 432, (b) 1440, (c) 480, (d) 6,350,400
9. (a) 126, (b) 40, (c) 20, (d) 1, (e) 121, (f) 45
10. (a) $\binom{12}{5} = 792$, (b) $\frac{1}{792}(0.1\%)$, (c) $\frac{7}{264}(2.7\%)$
11. (a) $\frac{9}{38}(23.7\%)$, (b) $\frac{69}{190}(36.3\%)$, (c) $\frac{121}{190}(0.3\%)(63.7\%)$