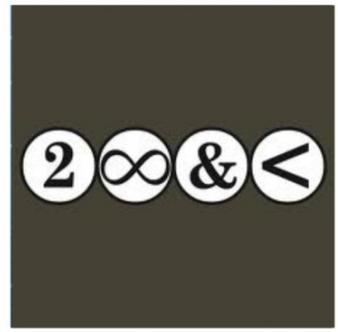
1.3 Properties of Limits - Part II

Investigate!



click if you are not a geek!

1. Determine each of the following limits graphically.

a) $\lim_{x\to\infty} \frac{6x^2-2x-1}{5x^2-x+1}$ $= 1.2$ $= \frac{6}{5}$

b) $\lim_{x\to\infty} \frac{7x^3-4}{3x^3+2x^2-1}$ $\doteq 2.333$ $= \frac{7}{3}$

c) $\lim_{x\to\infty} \frac{3x+1}{4x-5}$ $= 0.75$ $= \frac{3}{4}$

d) $\lim_{x\to\infty} \frac{4+2x+x^2}{1-x+2x^2}$ $= \frac{1}{2}$

- What pattern do you notice in the answers?

  Limit is equal to the leading coefficients

- What do each of these cases have in common?

  Degree numerator = Degree Denominator

- What is the value of $\lim_{x\to\infty} \frac{1}{x}$?

  $= 0$

- How can we use the fact that $\lim_{x\to\infty} \frac{1}{x} = 0$ to help determine these limits algebraically?

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method:
- divide all terms by the highest power of x in the denominator
- simplify
- evaluate the limit as $x \to \pm\infty$

- What does this limit represent graphically?

Value of horizontal asymptote

Ex. 1 Determine the following limit algebraically.

$$\lim_{x\to\infty} \frac{6x^2-5x+2}{-7x^2+3x} = \lim_{x\to\infty} \frac{\frac{6x^2}{x^2}-\frac{5x}{x^2}+\frac{2}{x^2}}{\frac{-7x^2}{x^2}+\frac{3x}{x^2}}$$

$$= \lim_{x\to\infty} \frac{6-\frac{5}{x}+\frac{2}{x^2}}{-7+\frac{3}{x}}$$

$$= \lim_{x\to\infty} \frac{6-0+0}{-7+0}$$

$$= -\frac{6}{7}$$

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2. Determine each of the following limits graphically.

a) $\lim_{x\to\infty} \frac{x^2+x+2}{4x^3-1}$ $= 0$

b) $\lim_{x\to\infty} \frac{1}{x+3}$ $= 0$

c) $\lim_{x\to\infty} \frac{2x}{x^2+2x-3}$ $= 0$

- What pattern do you notice in the answers?

  All equal to zero

- What do each of these cases have in common?

  Degree Numerator < Degree of denominator

3. Determine each of the following limits graphically.

a) $\lim_{x\to\infty} \frac{2x^3}{x^2+1}$ $= \infty$

b) $\lim_{x\to\infty} \frac{3x^4 - x}{x^2+1}$ $= \infty$

c) $\lim_{x\to\infty} \frac{6x^2}{5}$ $= \infty$

- What pattern do you notice in the answers?

All ∞

- What do each of these cases have in common?

Degree of numerator > Degree of denominator

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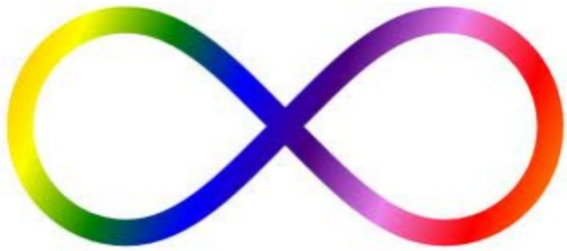
Summary...

for limits to infinity

1. degree of numerator = degree of denominator
   limit = ratio of leading coefficients

2. degree of numerator < degree of denominator
   limit = 0

3. degree of numerator > degree of denominator
   limit = $\pm\infty$



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4. Simplify and determine the limit without graphing.

a) $\lim_{x\to\infty} \dfrac{\frac{5}{x}}{\frac{6}{x}-\frac{1}{x^2}}$

Should be $= 5/6$

Degree?

$\lim_{x\to\infty} \dfrac{5x^{-1}}{6x^{-1}-x^{-2}}$

How to show?

$\lim_{x\to\infty} \dfrac{\frac{5}{x}}{\frac{6}{x}-\frac{1}{x^2}} \cdot \dfrac{x^2}{x^2}$

$= \lim_{x\to\infty} \dfrac{5x}{6x-1}$

$= \dfrac{5}{6}$

b) $\lim_{x\to\infty} \dfrac{\frac{3}{x^3}+\frac{2}{x^2}}{\frac{1}{x}} \cdot \dfrac{x^3}{x^3}$

$= \lim_{x\to\infty} \dfrac{3+2x}{x^2}$

$= 0$

(degree of denom > degree num.)

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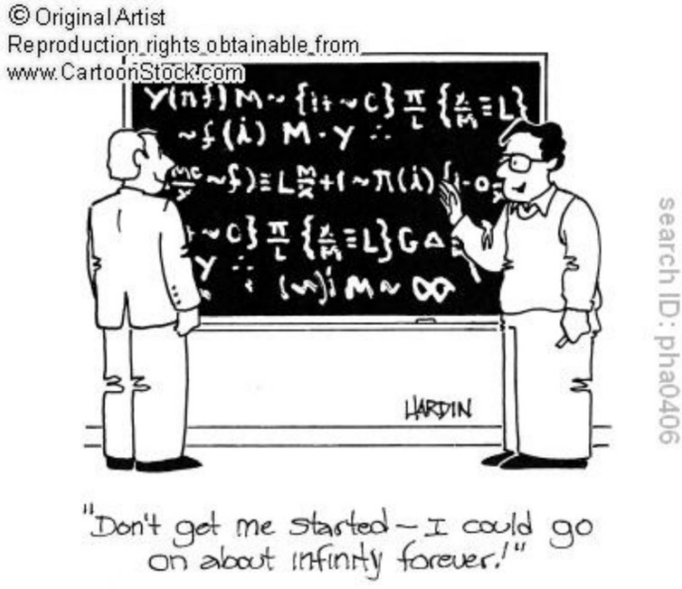
Homework
Handout "Limits to Infinity"



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