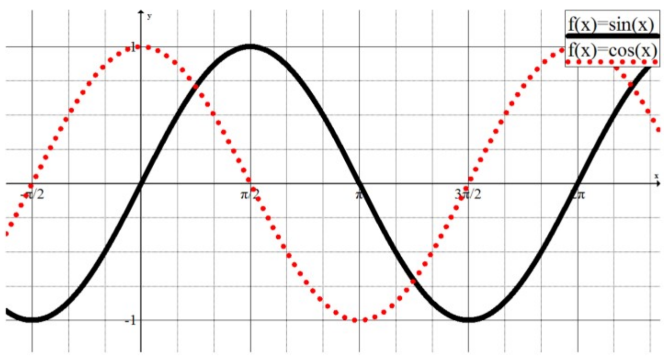
1.8 e^x & lnx

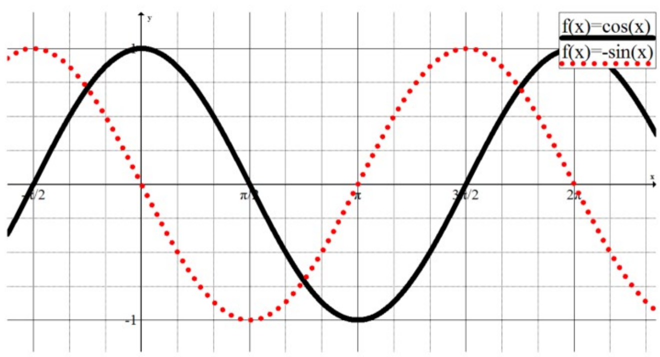
(don't take notes yet)

Summary from Investigating Trig and Exp Functions:

IF $f(x) = \sin x$ then $f'(x) = \cos x$



IF $f(x) = \cos x$ then $f'(x) = -\sin x$



Effect of Transformations on Base Trig Functions:

vertical translation - no effect on derivative

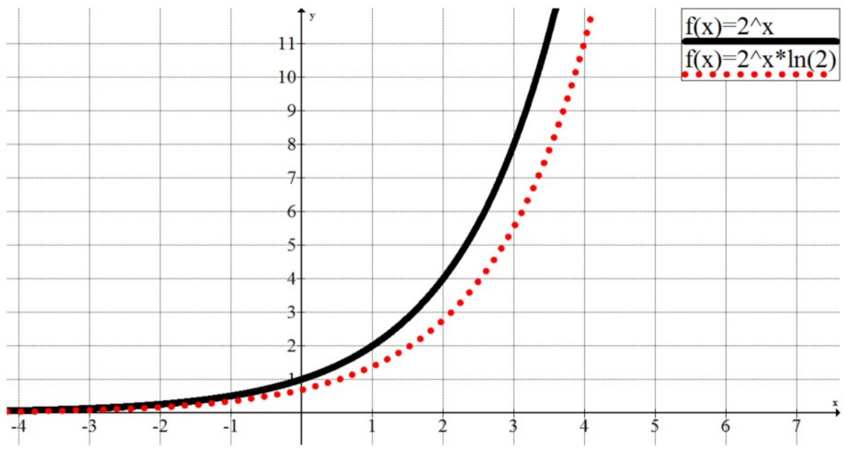
horizontal translation- derivative has same horizontal translation

vertical stretch/reflection - derivative has same vertical stretch/reflection

**horizontal stretch/reflection
-derivative has the same transformations BUT ALSO impacts the vertical stretch of the derivative function
-a shorter period results in steeper tangents, a longer period results in flatter tangents...this affects the amplitude of the derivative function

IF $f(x) = 2^x$, then $f'(x) = 2^x (k)$, where $k = \ln 2$.
This represents a vertical compression of $f(x)$.

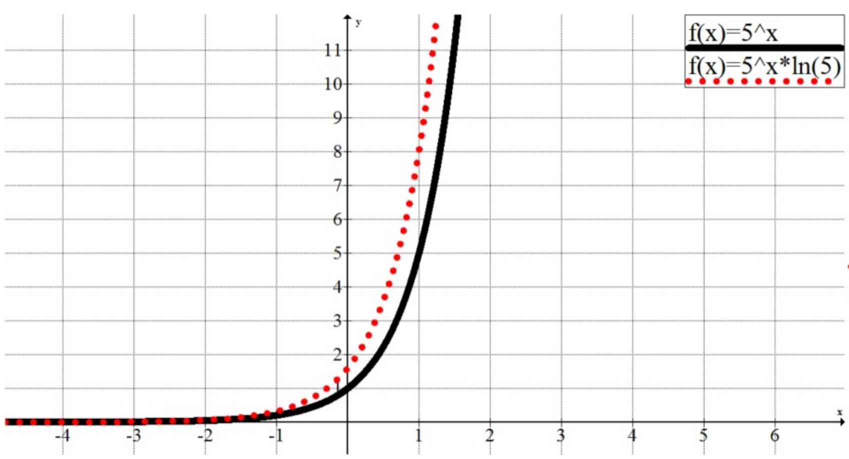
$$k = \frac{f'(x)}{f(x)} = \ln 2 \doteq 0.69$$



∴ $k < 1$
compression

IF $f(x) = 5^x$, then $f'(x) = 5^x (k)$, where $k = \ln 5$.
This represents a vertical stretch of $f(x)$.

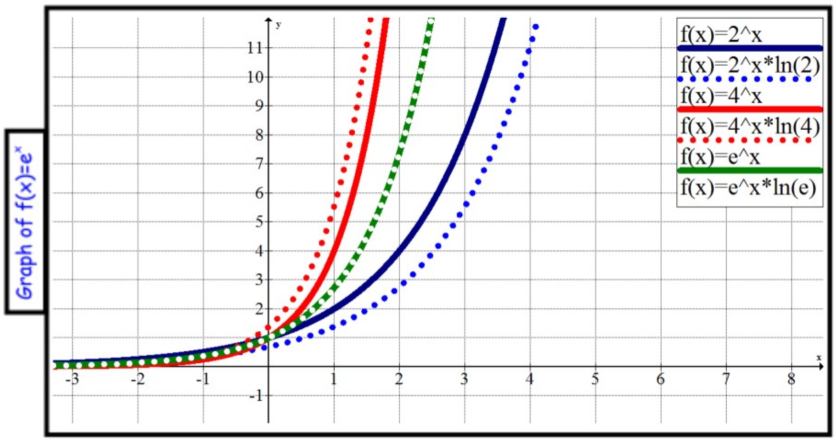
$$k = \frac{f'(x)}{f(x)} = \ln 5 \doteq 1.61$$



∴ $k > 1$
stretch

Q: If $f(x) = a^x$, is there a value of "a" where the derivative function would result in the same curve as the original function?
(ie. no stretch or compression)

Desmos Demo



When "a" is approximately equal to 2.72, the derivative function is equivalent to the original function.

When $f(x)=2.72^x$, the derivative function $f'(x)=2.72^x$ (same as f(x))
When the base is close to 2.72 the value of k approaches 1 ...therefore, there is no compression or stretch.

This is one of many definitions of the number "e". Called Euler's Number (thus the "e").

e = the base of an exponential function whose derivative function is itself

e = 2.71828182845904523536028747135266249775724709369995957 4966...

e is an irrational number (it cannot be written as a fraction, it never ends, there is no repeating pattern)

1.8 e^x & lnx

(now start taking notes!)

$\log_e$ is used so commonly we have a short form: *ln*

Log Laws:

$\log_a mn = \log_a m + \log_a n$

$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

$\log_a m^p = p(\log_a m)$

$\log_a 1 = 0$

$\log_a a^x = x$

$a^{\log_a x} = x$

Change of Base Formula:

recall: $\log_b a = \frac{\log_m a}{\log_m b}$

when m = e,

$\log_b a = \frac{\log_e a}{\log_e b}$

$= \frac{\ln a}{\ln b}$

Ex. 2 Simplify/evaluate each of the following.

a) $\log_5 1$ $= 0$

b) $\log_6 6^x$ $= x$

c) $6^{\log_6 x}$ $= x$

d) $\ln e$ $= 1$

e) $\ln 1$ $= 0$

f) $e^{\ln x}$ $= x$

g) $\ln e^x$ $= x$

Ex. 3 Simplify and evaluate each of the following.

a) $\log_6 2 + \log_6 3$

$= \log_6 6$

$= 1$

b) $\log_2 24 - \log_2\left(\frac{3}{4}\right)$

$= \log_2 \frac{24}{\frac{3}{4}}$

$= \log_2 32$

$= 5$

$24 \div \frac{3}{4}$

$= 24 \times \frac{4}{3}$

$= 32$

c) $2\log_2\sqrt{8} - 2\log_2 4$

$= 2\log_2 2^{\frac{3}{2}} - 2\log_2 2^2$

$= 2\left(\frac{3}{2}\right) - 2(2)$

$= -1$

d) $3\ln 2 - 3\ln 5$

$= 3(\ln 2 - \ln 5)$

$= 3\ln\frac{2}{5}$

$\doteq -2.7$

Ex. 4 Use your calculator to evaluate.

a) $\log_2 18$

$= \frac{\log 18}{\log 2}$

$\doteq 4.17$

b) $\log_5 3$

$\doteq 0.68$

c) $\log_e 10$

$\doteq 2.3$

$\ln 10$

$\doteq 2.3$

Homework: Handout

ONLY:
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pg 2: 1-8