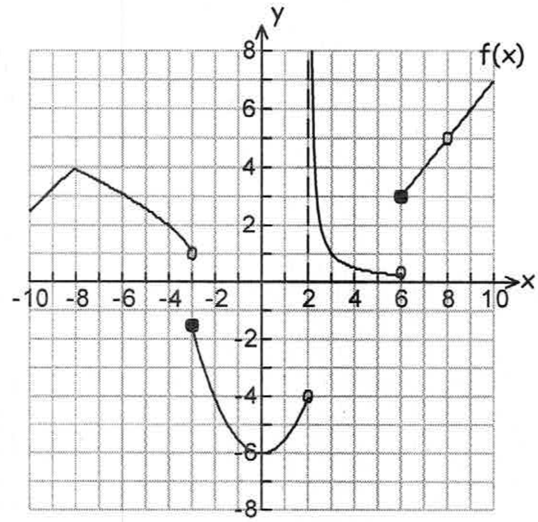
LIMIT Answers!

L	1. a) 4 b) 1 c) $+\infty$/DNE d) 1 e) 5 f) -1.5 g) DNE h)	2a) $y' = -2\cos x$ b) $f'(x) = -6\sin(2x)$ c) $g'(x) = 3^{x-2} \cdot \ln 3$ d) $y' = 2e^{2x}$
I	$y = (\frac{4}{9}\ln 3)x + \frac{4+8\ln 3}{9}$ GROSS! Also ok: $y \doteq 0.49x + 1.42$	
M	$(3, -5)$	
I_2	a) -64 b) $\frac{1}{36}$ c) 4 d) $\frac{1}{75}$	
T	a) 4 b) $\frac{1}{3}$ c) ∞ d) $-\frac{4}{5}$	e) 0 f) 0 g) DNE h) ∞

Station L

1. Complete the following based on the graph of f(x) to the right.



a) $\lim_{x \to -8} f(x) =$ 4

b) $\lim_{x \to -3^-} f(x) =$ 1

c) $\lim_{x \to 2^+} f(x) =$ $+\infty$ / DNE

d) $\lim_{x \to 3} f(x) =$ 1

e) $\lim_{x \to 8} f(x) =$ 5

f) f(-3)= -1.5

g) f(2)= DNE

2. State the derivative

a) $y = -2\sin x$
$y' = -2\cos x$

b) $f(x) = 3\cos(2x) - 3$
$f'(x) = -6\sin(2x)$

c) $g(x) = 3^{x-2} + 1$
$g'(x) = \ln 3 \cdot 3^{x-2}$

d) $y = e^{2x}$
$y' = 2e^{2x}$

STATION I

Determine the equation of the tangent to $y=4(3)^x$ at $x = -2$.

① Find slope @ $x = -2$
② Find equation of line through $(-2, f(-2))$

① $m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$= \lim_{h \to 0} \frac{4 \cdot 3^{-2+h} - 4 \cdot 3^{-2}}{h}$

$= 4 \cdot 3^{-2} \lim_{h \to 0} \frac{3^h - 1}{h}$

$= 4 \cdot 3^{-2} \cdot \ln 3$

$= \frac{4}{9} \ln 3$

② $f(-2) = 4 \cdot 3^{-2}$
$= \frac{4}{9}$

③ Equation through $(-2, \frac{4}{9})$ and
$m = \frac{4}{9} \ln 3$

$y = mx + b$

$\frac{4}{9} = \frac{4}{9} \ln 3 (-2) + b$

$b = \frac{4}{9} + \frac{8}{9} \ln 3$

$= \frac{4 + 8 \ln 3}{9}$

$\therefore y = \left(\frac{4}{9} \ln 3\right)x + \frac{4 + 8\ln 3}{9}$

GROSS!

also ok

$y \doteq 0.49x + 1.42$

Station M

At what point on the parabola $y = x^2 - 2x - 8$ is the tangent line parallel to the line $4x - y - 5 = 0$?

① Figure out target slope
② Find derivative
③ set derivative equal to target slope and solve

① $4x - y - 5 = 0$
$y = 4x - 5$
$m = 4$

② $$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{1}{h}\left[(x+h)^2 - 2(x+h) - \cancel{8} - x^2 + 2x + \cancel{8}\right]$$
$$= \lim_{h \to 0} \frac{1}{h}\left(\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h - \cancel{x^2} + \cancel{2x}\right)$$
$$= \lim_{h \to 0} \frac{1}{\cancel{h}}\left[\cancel{h}(2x + \overset{0}{\cancel{h}} - 2)\right]$$
$$= 2x - 2$$

③ $2x - 2 = 4$
$2x = 6$
$x = 3$

what point for $x = 3$?
$f(3) = 9 - 6 - 8$
$= -5$

$\therefore$ $(3, -5)$ has the parallel slope to the given line

Station I2

Evaluate each of the following limits.

a) $\lim\limits_{x\to -2}\frac{2x^4-32}{x+2} = \lim\limits_{x\to -2}\frac{2(x^4-16)}{(x+2)}$

$= \lim\limits_{x\to -2}\frac{2(x^2+4)(x-2)\cancel{(x+2)}}{\cancel{(x+2)}}$

$= 2[(-2)^2+4](-2-2)$

$= 2(8)(-4)$

$= -64$

b) $\lim\limits_{x\to 3}\frac{\sqrt{x+6}-3}{x^2-9}\cdot\frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}$

$= \lim\limits_{x\to 3}\frac{\cancel{x+6-9}}{\cancel{(x-3)}(x+3)(\sqrt{x+6}+3)}$

$= \lim\limits_{x\to 3}\frac{1}{(x+3)(\sqrt{x+6}+3)}$

$= \frac{1}{6(3+3)} = \frac{1}{36}$

c) $\lim\limits_{x\to 3} f(x), f(x)=\begin{cases} x^2-5, & x\le 3 \\ \sqrt{x+13}, & x>3\end{cases}$

LH

$\lim\limits_{x\to 3^-} x^2-5$

$= 9-5$

$= 4$

RH

$\lim\limits_{x\to 3^+}\sqrt{x+13}$

$= \sqrt{16}$

$= 4$

$\therefore \lim\limits_{x\to 3} f(x) = 4$

(since LH = RH)

d) $\lim\limits_{x\to 0}\frac{(x+125)^{\frac{1}{3}}-5}{x}$

let $u=(x+125)^{\frac{1}{3}}$

$\therefore u^3 = x+125$

$x = u^3-125$

as $x\to 0$

$u\to 5$

$\therefore \lim\limits_{u\to 5}\frac{u-5}{u^3-125}$

$= \lim\limits_{u\to 5}\frac{\cancel{u-5}}{\cancel{(u-5)}(u^2+5u+25)}$

$= \frac{1}{25+25+25}$

$= \frac{1}{75}$

Station T

Find the limits

a) $\lim_{x\to 0}\frac{x^2+8}{x+2}$ $=4$

b) $\lim_{x\to 0}\frac{x+x^2}{3x}$

c) $\lim_{x\to \infty}\frac{x^2+x-6}{x-2}$ $=\infty$

d) $\lim_{x\to \infty}\frac{4x^3-5x+7}{x-5x^3-1}$ $=-\frac{4}{5}$

e) $\lim_{x\to \infty}\left(\frac{2}{3}\right)^x$ $=0$

f) $\lim_{x\to \infty}\frac{5x^2+8x}{3x^3-1}$ $=0$

g) $\lim_{x\to -5}\sqrt{x+5}$ $=$ DNE
(NO LH Limit

h) $\lim_{x\to \infty}\left(x+\frac{10}{x}\right)$

---

b) $\lim_{x\to 0}\frac{\cancel{x}(1+x)}{3\cancel{x}}$

$=\frac{1+0}{3}$

$=\frac{1}{3}$

h) $\lim_{x\to\infty}\left[\frac{x^2}{x}+\frac{10}{x}\right]$

$=\lim_{x\to\infty}\left(\frac{x^2+10}{x}\right)$

$=\infty$