

3.3 Critical Points, Local Maxima/Minima

Critical Values for $f'(x)$:

- include all values of x where $f'(x) = 0$ OR $f'(x)$ is undefined
- all local max/mins occur at a critical value of $f'(x)$
- not all critical values yield a max/min (do 1st derivative test)
- critical values where $f'(x)$ is undefined give the locations of cusps, corners, vertical tangents or a discontinuity of $f(x)$

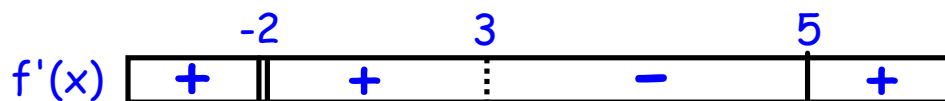
1st Derivative Test:

If $x=c$ is a critical value, then there is a:

- local maximum if $f'(x)$ changes from positive to negative at c
- local minimum if $f'(x)$ changes from negative to positive at c

Eg. 1

Given this strip for $f'(x)$, identify:



critical values: $-2, 3, 5$

intervals of increase: $(-\infty, -2) \cup (-2, 3) \cup (5, \infty)$

intervals of decrease: $(3, 5)$

local max: None

local min: $x = 5$

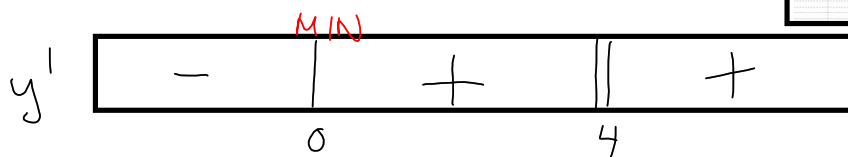
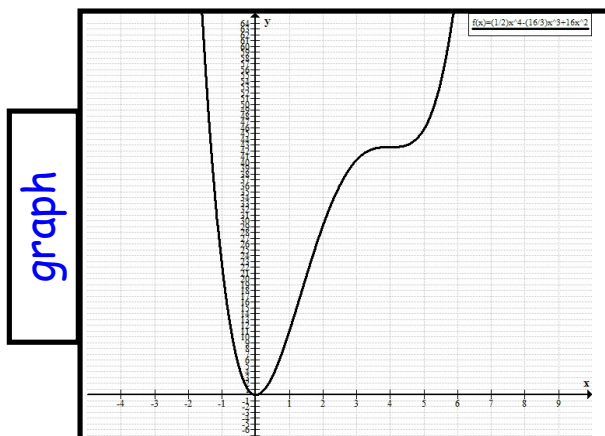
Asymptote: $x = 3$

Ex. 2 Determine the critical values and local extrema.

a) $y = \frac{1}{2}x^4 - \frac{16}{3}x^3 + 16x^2$

$$\begin{aligned} y' &= 2x^3 - 16x^2 + 32x \\ &= 2x(x^2 - 8x + 16) \\ &= 2x(x-4)^2 \end{aligned}$$

Crit #s: $x = 0, 4$



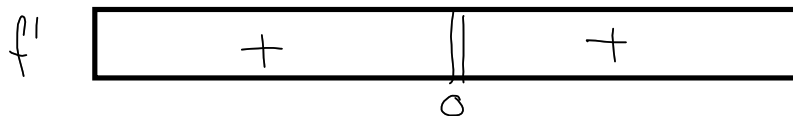
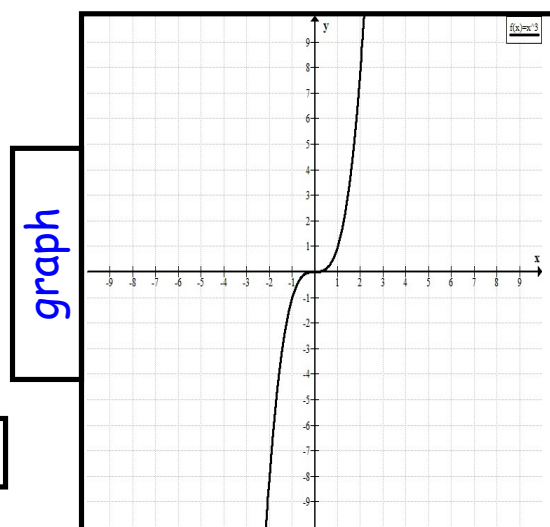
\therefore Min @ $x=0$ $(0, 0)$

b) $y = x^3$

$$y' = 3x^2$$

Crit #s:

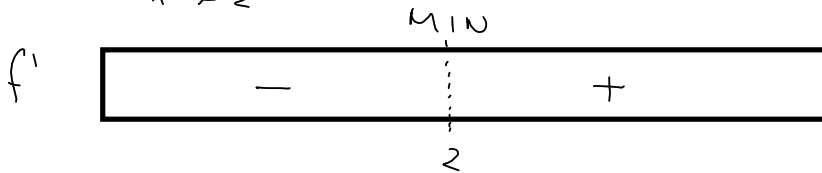
$x = 0$



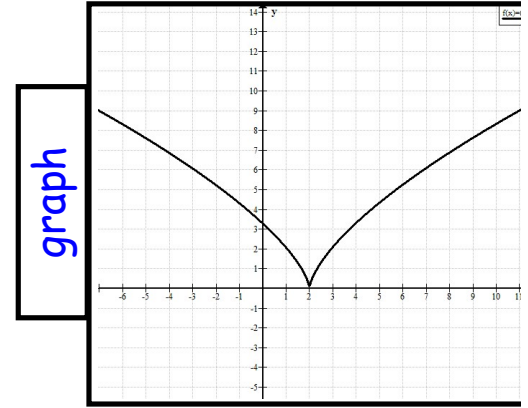
c) $f(x) = (3x-6)^{\frac{2}{3}}$
 $f'(x) = \frac{2}{3}(3x-6)^{-\frac{1}{3}}(3)$
 $= \frac{2}{(3x-6)^{\frac{1}{3}}}$

Crit #s:

$x \neq 2$



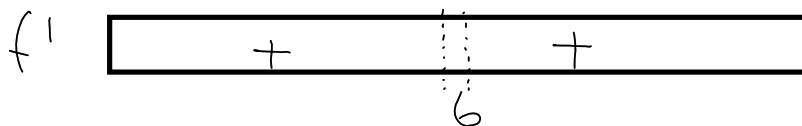
\therefore MIN @ $x=2$, even though $f'(2)$ DNE
 $(2,0)$



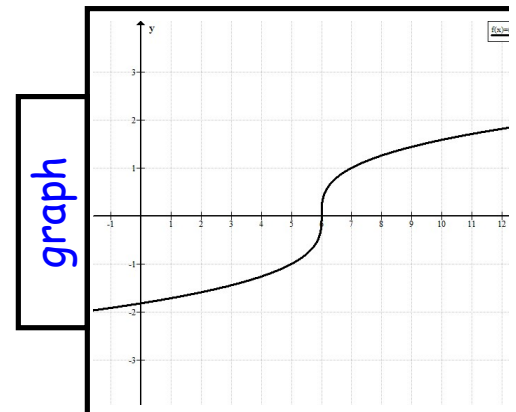
d) $f(x) = \sqrt[3]{x-6}$
 $= (x-6)^{\frac{1}{3}}$
 $f'(x) = \frac{1}{3}(x-6)^{-\frac{2}{3}}$
 $= \frac{1}{3(x-6)^{\frac{2}{3}}}$

Crit #s:

$x \neq 6$



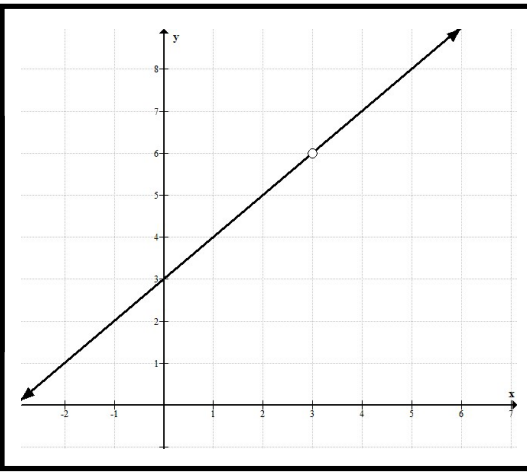
\therefore No MAX/MIN



$$\begin{aligned}
 e) \quad f(x) &= \frac{x^2 - 9}{x - 3} \\
 &= \frac{(x-3)(x+3)}{x-3} \\
 &= x+3, \quad x \neq 3
 \end{aligned}$$

$$\begin{aligned}
 f'(x) &= 1 \\
 \text{Crit \#s: } &x \neq 3
 \end{aligned}$$

graph



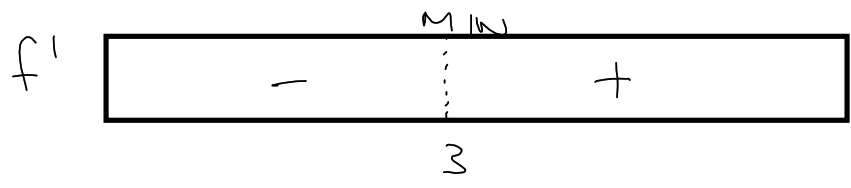
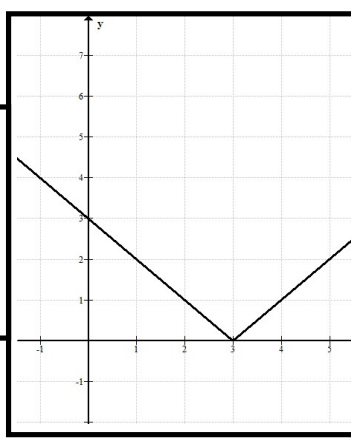
$$f) \quad f(x) = |x - 3|$$

ALWAYS BREAK INTO TWO PARTS

$$f(x) = \begin{cases} x-3, & x > 3 \\ -x+3, & x < 3 \end{cases}$$

$$f'(x) = \begin{cases} 1, & x > 3 \\ -1, & x < 3 \end{cases} \quad f'(3) \text{ DNE}$$

graph



$\therefore \text{Min@ } x=3$
 $(3, 0)$

Homework
page 179
5ac, 7ace, 9, 10, 12,

