

3.6 Curve Sketching (Day 1)

Analysis of $f(x)$

1. Domain
2. Intercepts(x and y)
3. Asymptotes (vertical, horizontal, oblique)

Analysis of $f'(x)$

4. Intervals of increase/decrease

5. Local extrema (max/min) →

Can use the 1st or 2nd derivative test to check if max/min

Analysis of $f''(x)$

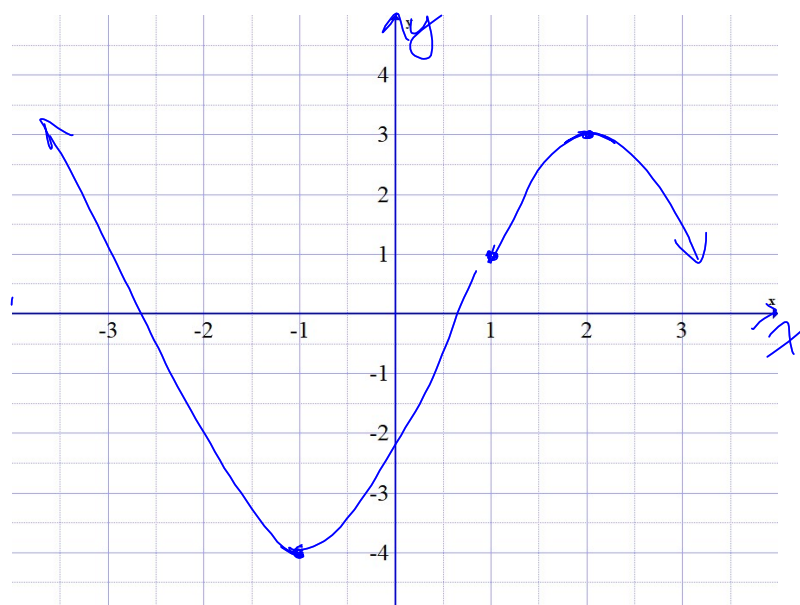
6. Intervals of concavity
7. Points of inflection

Last step:

8. SKETCH!

Ex. 1 Sketch, given the following:

- Local max at $(2,3)$ and local min at $(-1,-4)$
- Decreasing on $(-\infty, -1)$ and $(2, \infty)$; increasing on $(-1, 2)$
- CU on $(-\infty, 1)$; CD on $(1, \infty)$
- POI at $(1, 1)$



Ex. 2 Sketch $y = x^3 - 6x^2 + 3x + 10$

y

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 3 & 10 \\ & & -1 & 7 & -10 \\ \hline & 1 & -7 & 10 & 0 \end{array}$$

$(x+1)$ Divided evenly!

$$y = (x+1)(x^2 - 7x + 10)$$

$$= (x+1)(x-5)(x-2)$$

$$y' = 3x^2 - 12x + 3$$

$$0 = 3x^2 - 12x + 3$$

$$= 3(x^2 - 4x + 1)$$

QUAD FORM

$$x = 2 \pm \sqrt{3}$$

$$= 0.3, 3.7$$

$$f(0.3) = 10.4$$

$$f(3.7) = -10.4$$

$$y'' = 6x - 12$$

$$0 = 6x - 12$$

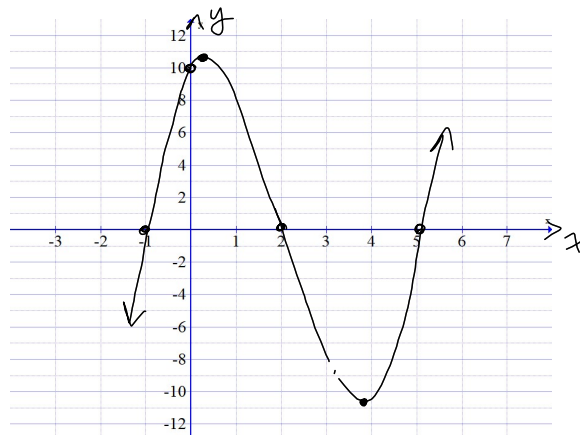
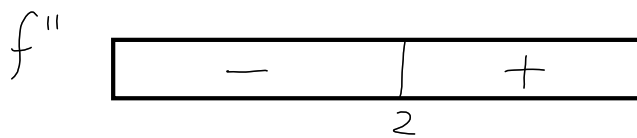
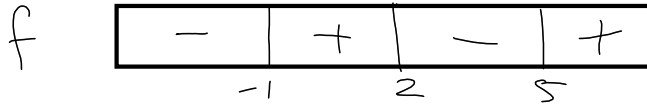
$$x = 2$$

$$f(2) = 0$$

Crit.#s:

Zeros: -1, 2, 5

y-int: 10



Ex. 3 Sketch $y = \frac{x^2}{1-x^2}$

$$y = \frac{x^2}{1-x^2}$$

$$y' = \frac{2x(1-x^2) - x^2(-2x)}{(1-x^2)^2}$$

Zeros: 0, 0

VA: $x = \pm 1$

HA: $y = -1$

$$= \frac{2x - 2x^3 + 2x^3}{(1-x^2)^2}$$

$$= \frac{2x}{(1-x^2)^2}$$

Crit #s:

$$x = 0$$

$$x \neq \pm 1, \pm 1$$

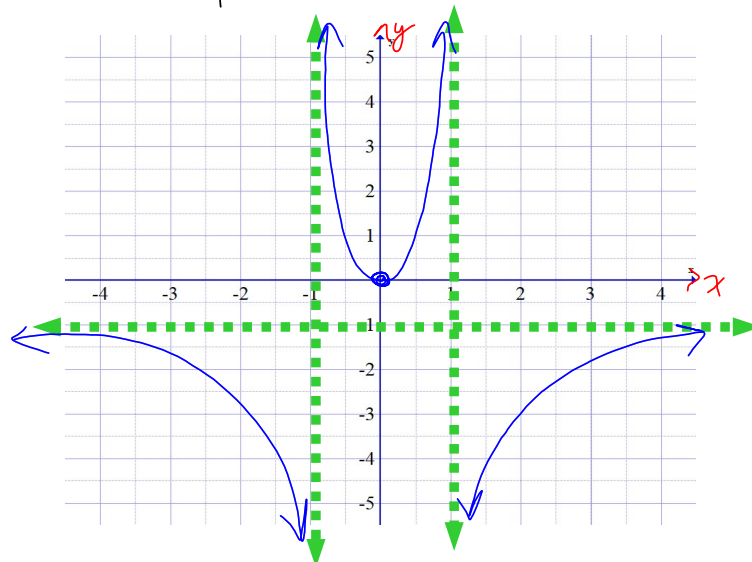
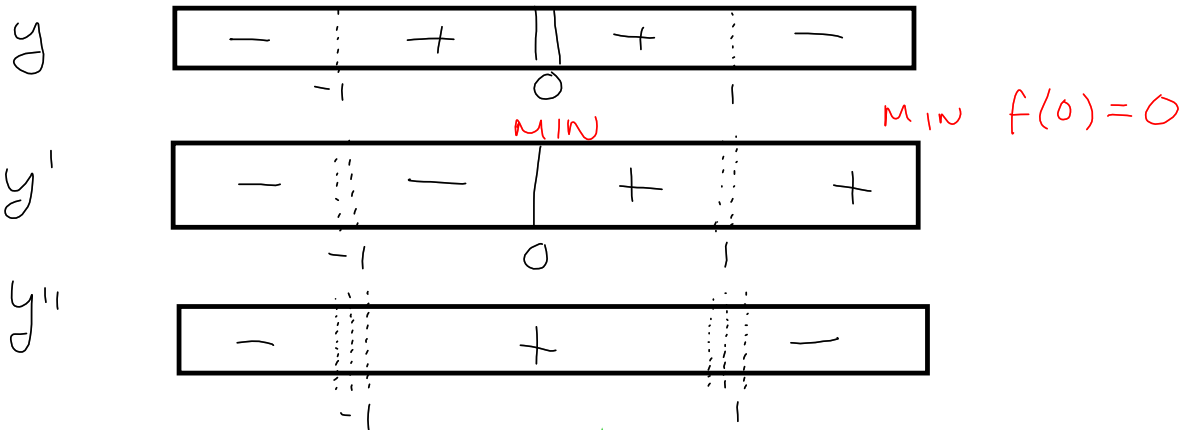
$$y'' = \frac{2(1-x^2)^2 - 2x(2)(1-x^2)(-2x)}{(1-x^2)^{4+3}}$$

$$= \frac{2 - 2x^2 + 8x^2}{(1-x^2)^3}$$

$$= \frac{6x^2 + 2}{(1-x^2)^3}$$

Crit #s:

$$x \neq \pm 1, \pm 1, \pm 1$$



Ex. 4 Sketch $y = \frac{x^2 + 2x - 8}{x - 3}$

$$y = \frac{(x-2)(x+4)}{(x-3)} \quad y' = \frac{(2x+2)(x-3) - (x^2+2x-8)(1)}{(x-3)^2}$$

$$= \frac{2x^2 - 6x + 2x - 6 - x^2 - 2x + 8}{(x-3)^2}$$

$$= \frac{x^2 - 6x + 2}{(x-3)^2}$$

Zeros: 2, -4
 VA: $x = 3$
 y-int: $y = \frac{8}{3} = 2.66$

OA? $x+5$
 $x-3 \overline{) x^2+2x-8}$
 $\underline{x^2-3x}$

$\underline{5x-8}$
 $\underline{5x-15}$

OA: $y = x+5$

Crit #s

$x = 3 \pm \sqrt{7}$
 $= 0.4, 5.6$

$x \neq 3, 3$

$y'' = \frac{(2x-6)(x-3)^2 - (x^2-6x+2)(2)(x-3)(1)}{(x-3)^4}$

$$= \frac{2x^2 - 6x - 6x + 18 - 2x^2 + 12x - 4}{(x-3)^3}$$

$= \frac{14}{(x-3)^3}$

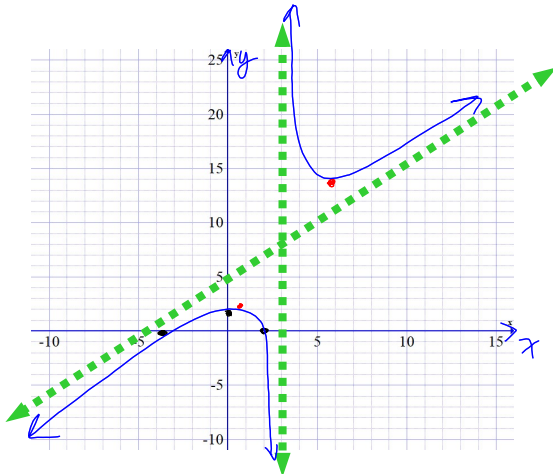
Crit #s

$x \neq 3, 3, 3$

y	-	+	-	+
		-4	2	3

y'	+	-	-	+
		0.4	3	5.6

y''	-	+
		3



Max $f(0.4) = 2.7$
 Min $f(5.6) = 13.3$

Homework
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