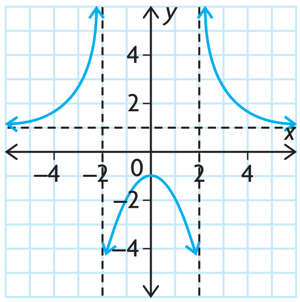
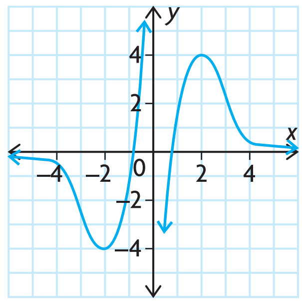
Exercise 4.3

PART A

1. State the equations of the vertical and horizontal asymptotes of the curves shown.

a. 

b. 

C 2. Under what conditions does a rational function have vertical, horizontal, and oblique asymptotes?

3. Evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$, using the symbol "∞" when appropriate.

a. $f(x) = \dfrac{2x + 3}{x - 1}$

b. $f(x) = \dfrac{5x^2 - 3}{x^2 + 2}$

c. $f(x) = \dfrac{-5x^2 + 3x}{2x^2 - 5}$

d. $f(x) = \dfrac{2x^5 - 3x^2 + 5}{3x^4 + 5x - 4}$

4. For each of the following, check for discontinuities and state the equation of any vertical asymptotes. Conduct a limit test to determine the behaviour of the curve on either side of the asymptote.

a. $y = \dfrac{x}{x + 5}$

b. $f(x) = \dfrac{x + 2}{x - 2}$

c. $s = \dfrac{1}{(t - 3)^2}$

d. $y = \dfrac{x^2 - x - 6}{x - 3}$

e. $f(x) = \dfrac{6}{(x + 3)(x - 1)}$

f. $y = \dfrac{x^2}{x^2 - 1}$

5. For each of the following, determine the equations of any horizontal asymptotes. Then state whether the curve approaches the asymptote from above or below.

a. $y = \dfrac{x}{x + 4}$

b. $f(x) = \dfrac{2x}{x^2 - 1}$

c. $g(t) = \dfrac{3t^2 + 4}{t^2 - 1}$

d. $y = \dfrac{3x^2 - 8x - 7}{x - 4}$

PART B

K 6. For each of the following, check for discontinuities and then use at least two other tests to make a rough sketch of the curve. Verify using a calculator.

a. $y = \frac{x - 3}{x + 5}$

b. $f(x) = \frac{5}{(x + 2)^2}$

c. $g(t) = \frac{t^2 - 2t - 15}{t - 5}$

d. $y = \frac{(2 + x)(3 - 2x)}{(x^2 - 3x)}$

7. Determine the equation of the oblique asymptote for each of the following:

a. $f(x) = \frac{3x^2 - 2x - 17}{x - 3}$

b. $f(x) = \frac{2x^2 + 9x + 2}{2x + 3}$

c. $f(x) = \frac{x^3 - 1}{x^2 + 2x}$

d. $f(x) = \frac{x^3 - x^2 - 9x + 15}{x^2 - 4x + 3}$

8. a. For question 7, part a., determine whether the curve approaches the asymptote from above or below.

b. For question 7, part b., determine the direction from which the curve approaches the asymptote.

9. For each function, determine any vertical or horizontal asymptotes and describe its behaviour on each side of any vertical asymptote.

a. $f(x) = \frac{3x - 1}{x + 5}$

b. $g(x) = \frac{x^2 + 3x - 2}{(x - 1)^2}$

c. $h(x) = \frac{x^2 + x - 6}{x^2 - 4}$

d. $m(x) = \frac{5x^2 - 3x + 2}{x - 2}$

A 10. Use the algorithm for curve sketching to sketch the graph of each function.

a. $f(x) = \frac{3 - x}{2x + 5}$

b. $h(t) = 2t^3 - 15t^2 + 36t - 10$

c. $y = \frac{20}{x^2 + 4}$

d. $s(t) = t + \frac{1}{t}$

e. $g(x) = \frac{2x^2 + 5x + 2}{x + 3}$

f. $s(t) = \frac{t^2 + 4t - 21}{t - 3}, t \geq -7$

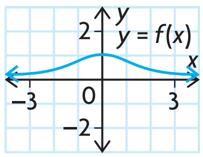
11. Consider the function $y = \frac{ax + b}{cx + d}$, where a, b, c, and d are constants, $a \neq 0, c \neq 0$.

a. Determine the horizontal asymptote of the graph.

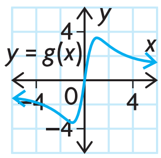
b. Determine the vertical asymptote of the graph.

12. Use the features of each function's graph to sketch the graph of its first derivative.

a.

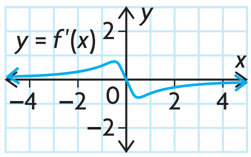


b.

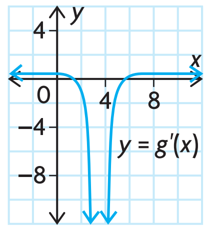


13. A function’s derivative is shown in each graph. Use the graph to sketch a possible graph for the original function.

a.



b.



14. Let $f(x) = \frac{-x - 3}{x^2 - 5x - 14}$, $g(x) = \frac{x - x^3}{x - 3}$, $h(x) = \frac{x^3 - 1}{x^2 + 4}$, and $r(x) = \frac{x^2 + x - 6}{x^2 - 16}$. How can you can tell from its equation which of these functions has

a. a horizontal asymptote?

b. an oblique asymptote?

c. no vertical asymptote?

Explain. Determine the equations of all asymptote(s) for each function. Describe the behaviour of each function close to its asymptotes.

PART C

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15. Find constants a and b such that the graph of the function defined by $f(x) = \frac{ax + 5}{3 - bx}$ will have a vertical asymptote at $x = 5$ and a horizontal asymptote at $y = -3$.

16. To understand why we cannot work with the symbol ∞ as though it were a real number, consider the functions $f(x) = \frac{x^2 + 1}{x + 1}$ and $g(x) = \frac{x^2 + 2x + 1}{x + 1}$.

a. Show that $\lim_{x \to +\infty} f(x) = +\infty$ and $\lim_{x \to +\infty} g(x) = +\infty$.

b. Evaluate $\lim_{x \to +\infty} [f(x) - g(x)]$, and show that the limit is not zero.

17. Use the algorithm for curve sketching to sketch the graph of the function $f(x) = \frac{2x^2 - 2x}{x^2 - 9}$.