IN SUMMARY

Key Ideas

- In R^3, $\vec{i} = (1, 0, 0)$, $\vec{j} = (0, 1, 0)$, $\vec{k} = (0, 0, 1)$. All are unit vectors along the x-, y- and z-axes, respectively.
- $\overrightarrow{OP} = (a, b, c) = a\vec{i} + b\vec{j} + c\vec{k}$, $|\overrightarrow{OP}| = \sqrt{a^2 + b^2 + c^2}$
- The vector between two points with its tail at $A(x_1, y_1, z_1)$ and head at $B(x_1, y_1, z_1)$ is determined as follows:
  $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (x_2, y_2, z_2) - (x_1, y_1, z_1) = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$
- The vector $\overrightarrow{AB}$ is equivalent to the position vector $\overrightarrow{OP}$ since their directions and magnitude are the same.
  $|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Need to Know

- If $\overrightarrow{OA} = (a, b, c)$ and $\overrightarrow{OD} = (d, e, f)$, then $\overrightarrow{OA} + \overrightarrow{OD} = (a + d, b + e, c + f)$.
- $m\overrightarrow{OP} = m(a, b, c) = (ma, mb, mc)$, $m \in \mathbf{R}$

Exercises 6.7

PART A

1. a. Write the vector $\overrightarrow{OA} = (-1, 2, 4)$ using the standard unit vectors.
   b. Determine $|\overrightarrow{OA}|$.
2. Write the vector $\overrightarrow{OB} = 3\vec{i} + 4\vec{j} - 4\vec{k}$ in component form and calculate its magnitude.
3. If $\vec{a} = (1, 3, -3)$, $\vec{b} = (-3, 6, 12)$, and $\vec{c} = (0, 8, 1)$, determine $\left|\vec{a} + \frac{1}{3}\vec{b} - \vec{c}\right|$.
4. For the vectors $\overrightarrow{OA} = (-3, 4, 12)$ and $\overrightarrow{OB} = (2, 2, -1)$, determine the following:
   a. the components of vector $\overrightarrow{OP}$, where $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$
   b. $|\overrightarrow{OA}|$, $|\overrightarrow{OB}|$, and $|\overrightarrow{OP}|$
   c. $\overrightarrow{AB}$ and $|\overrightarrow{AB}|$. What does $\overrightarrow{AB}$ represent?

PART B

K

5. Given $\vec{x} = (1, 4, -1)$, $\vec{y} = (1, 3, -2)$, and $\vec{z} = (-2, 1, 0)$, determine a vector equivalent to each of the following:
   a. $\vec{x} - 2\vec{y} - \vec{z}$
   b. $-2\vec{x} - 3\vec{y} + \vec{z}$
   c. $\frac{1}{2}\vec{x} - \vec{y} + 3\vec{z}$
   d. $3\vec{x} + 5\vec{y} + 3\vec{z}$

6. Given $\vec{p} = 2\vec{i} - \vec{j} + \vec{k}$ and $\vec{q} = -\vec{i} - \vec{j} + \vec{k}$, determine the following in terms of the standard unit vectors.
   a. $\vec{p} + \vec{q}$ b. $\vec{p} - \vec{q}$ c. $2\vec{p} - 5\vec{q}$ d. $-2\vec{p} + 5\vec{q}$

7. If $\vec{m} = 2\vec{i} - \vec{k}$ and $\vec{n} = -2\vec{i} + \vec{j} + 2\vec{k}$, calculate each of the following:
   a. $|\vec{m} - \vec{n}|$ b. $|\vec{m} + \vec{n}|$ c. $|2\vec{m} + 3\vec{n}|$ d. $|-5\vec{m}|$

8. Given $\vec{x} + \vec{y} = -\vec{i} + 2\vec{j} + 5\vec{k}$ and $\vec{x} - \vec{y} = 3\vec{i} + 6\vec{j} - 7\vec{k}$, determine $\vec{x}$ and $\vec{y}$.

C

9. Three vectors, $\overrightarrow{OA} = (a, b, 0)$, $\overrightarrow{OB} = (a, 0, c)$, and $\overrightarrow{OC} = (0, b, c)$, are given.
   a. In a sentence, describe what each vector represents.
   b. Write each of the given vectors using the standard unit vectors.
   c. Determine a formula for each of $|\overrightarrow{OA}|$, $|\overrightarrow{OB}|$, and $|\overrightarrow{OC}|$.
   d. Determine $\overrightarrow{AB}$. What does $\overrightarrow{AB}$ represent?

10. Given the points $A(-2, -6, 3)$ and $B(3, -4, 12)$, determine each of the following:
    a. $|\overrightarrow{OA}|$ c. $\overrightarrow{AB}$ e. $\overrightarrow{BA}$
    b. $|\overrightarrow{OB}|$ d. $|\overrightarrow{AB}|$ f. $|\overrightarrow{BA}|$

11. The vertices of quadrilateral $ABCD$ are given as $A(0, 3, 5)$, $B(3, -1, 17)$, $C(7, -3, 15)$, and $D(4, 1, 3)$. Prove that $ABCD$ is a parallelogram.

12. Given $2\vec{x} + \vec{y} - 2\vec{z} = \vec{0}$, $\vec{x} = (-1, b, c)$, $\vec{y} = (a, -2, c)$, and $\vec{z} = (-a, 6, c)$, determine the value of the unknowns.

A

13. A parallelepiped is determined by the vectors $\overrightarrow{OA} = (-2, 2, 5)$, $\overrightarrow{OB} = (0, 4, 1)$, and $\overrightarrow{OC} = (0, 5, -1)$.
    a. Draw a sketch of the parallelepiped formed by these vectors.
    b. Determine the coordinates of all of the vertices for the parallelepiped.

T

14. Given the points $A(-2, 1, 3)$ and $B(4, -1, 3)$, determine the coordinates of the point on the x-axis that is equidistant from these two points.

PART C

15. Given $|\vec{a}| = 3$, $|\vec{b}| = 5$, and $|\vec{a} + \vec{b}| = 7$, determine $|\vec{a} - \vec{b}|$.