IN SUMMARY

Key Ideas

- The vector equation of a line in R^2 is $\vec{r} = \vec{r_0} + t\vec{m}$, $t \in \mathbf{R}$, where $\vec{m} = (a, b)$ is the direction vector and $\vec{r_0}$ is the vector from the origin to any point on the line whose general coordinates are (x_0, y_0). This is equivalent to the equation $(x, y) = (x_0, y_0) + t(a, b)$.
- The parametric form of the equation of a line is $x = x_0 + ta$ and $y = y_0 + tb$, $t \in \mathbf{R}$.

Need to Know

- In both the vector and parametric equations, t is a parameter. Every real number for t generates a different point that lies on the line.

Exercise 8.1

PART A

1. A vector equation is given as $\vec{r} = \left(\frac{1}{2}, -\frac{3}{4}\right) + s\left(\frac{1}{3}, \frac{1}{6}\right), s \in \mathbf{R}$. Explain why $\vec{m} = (-2, -1)$, $\vec{m} = (2, 1)$, and $\vec{m} = \left(\frac{2}{7}, \frac{1}{7}\right)$ are acceptable direction vectors for this line.
2. Parametric equations of a line are $x = 1 + 3t$ and $y = 5 - 2t$, $t \in \mathbf{R}$.
   a. Write the coordinates of three points on this line.
   b. Show that the point $P(-14, 15)$ lies on the given line by determining the parameter value of t corresponding to this point.
3. Identify the direction vector and a point on each of the following lines:
   a. $\vec{r} = (3, 4) + t(2, 1), t \in \mathbf{R}$
   b. $x = 1 + 2t, y = 3 - 7t, t \in \mathbf{R}$
   c. $\vec{r} = (4, 1 + 2t), t \in \mathbf{R}$
   d. $x = -5t, y = 6, t \in \mathbf{R}$

PART B

4. A line passes through the points $A(2, 1)$ and $B(-3, 5)$. Write two different vector equations for this line.
5. A line is defined by the parametric equations $x = -2 - t$ and $y = 4 + 2t, t \in \mathbf{R}$.
   a. Does $R(-9, 18)$ lie on this line? Explain.
   b. Write a vector equation for this line using the given parametric equations.
   c. Write a second vector equation for this line, different from the one you wrote in part b.

6. a. If the equation of a line is $\vec{r} = s(3, 4), s \in \mathbf{R}$, name the coordinates of three points on this line.
   b. Write a vector equation, different from the one given, in part a., that also passes through the origin.
   c. Describe how the line with equation $\vec{r} = (9, 12) + t(3, 4), t \in \mathbf{R}$ relates to the line given in part a.

C 7. A line has $\vec{r} = \left(\frac{1}{3}, \frac{1}{7}\right) + p(-2, 3), p \in \mathbf{R}$, as its vector equation. A student decides to "simplify" this equation by clearing the fractions and multiplies the vector $\left(\frac{1}{3}, \frac{1}{7}\right)$ by 21. The student obtains $\vec{r} = (7, 3) + p(-2, 3), p \in \mathbf{R}$, as a "correct" form of the line. Explain why multiplying a point in this way is incorrect.

8. A line passes through the points $Q(0, 7)$ and $R(0, 9)$.
   a. Sketch this line.
   b. Determine vector and parametric equations for this line.

K 9. A line passes through the points $M(4, 5)$ and $N(9, 5)$.
   a. Sketch this line.
   b. Determine vector and parametric equations for this line.

10. For the line $L: \vec{r} = (1, -5) + s(3, 5), s \in \mathbf{R}$, determine the following:
   a. an equation for the line perpendicular to L, passing through $P(2, 0)$
   b. the point at which the line in part a. intersects the y-axis

A 11. The parametric equations of a line are given as $x = -10 - 2s$, $y = 8 + s$, $s \in \mathbf{R}$. This line crosses the x-axis at the point with coordinates $A(a, 0)$ and crosses the y-axis at the point with coordinates $B(0, b)$. If O represents the origin, determine the area of the triangle AOB.

T 12. A line has $\vec{r} = (1, 2) + s(-2, 3), s \in \mathbf{R}$, as its vector equation. On this line, the points A, B, C, and D correspond to parametric values $s = 0$, 1, 2, and 3, respectively. Show that each of the following is true:
   a. $\overrightarrow{AC} = 2\overrightarrow{AB}$ b. $\overrightarrow{AD} = 3\overrightarrow{AB}$ c. $\overrightarrow{AC} = \frac{2}{3}\overrightarrow{AD}$

PART C

13. The line L has $x = 2 + t$, $y = 9 + t$, $t \in \mathbf{R}$, as its parametric equations. If L intersects the circle with equation $x^2 + y^2 = 169$ at points A and B, determine the following:
   a. the coordinates of points A and B
   b. the length of the chord AB

14. Are the lines $2x - 3y + 15 = 0$ and $(x, y) = (1, 6) + t(6, 4)$ parallel? Explain.