IN SUMMARY

Key Idea

- The Cartesian equation of a plane in R^3 is $Ax + By + Cz + D = 0$, where $\vec{n} = (A, B, C)$ is a normal to the plane and $\vec{n} = \vec{a} \times \vec{b}$. $\vec{a}$ and $\vec{b}$ are any two noncollinear direction vectors of the plane.

Need to Know

- Two planes whose normals are $\vec{n_1}$ and $\vec{n_2}$
  - are parallel if and only if $\vec{n_1} = k\vec{n_2}$ for any nonzero real number k.
  - are perpendicular if and only if $\vec{n_1} . \vec{n_2} = 0$.
  - have an angle θ between the planes determined by $\theta = \cos^{-1}\left(\frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right)$.

Exercise 8.5

PART A

1. A plane is defined by the equation $x - 7y - 18z = 0$.
   a. What is a normal vector to this plane?
   b. Explain how you know that this plane passes through the origin.
   c. Write the coordinates of three points on this plane.
2. A plane is defined by the equation $2x - 5y = 0$.
   a. What is a normal vector to this plane?
   b. Explain how you know that this plane passes through the origin.
   c. Write the coordinates of three points on this plane.
3. A plane is defined by the equation $x = 0$.
   a. What is a normal vector to this plane?
   b. Explain how you know that this plane passes through the origin.
   c. Write the coordinates of three points on this plane.
4. a. A plane is determined by a normal, $\vec{n} = (15, 75, -105)$, and passes through the origin. Write the Cartesian equation of this plane, where the normal is in reduced form.
   b. A plane has a normal of $\vec{n} = \left(-\frac{1}{2}, \frac{3}{4}, \frac{7}{16}\right)$ and passes through the origin. Determine the Cartesian equation of this plane.

PART B

5. A plane is determined by a normal, $\vec{n} = (1, 7, 5)$, and contains the point $P(-3, 3, 5)$. Determine a Cartesian equation for this plane using the *two* methods shown in Example 1.

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6. The three noncollinear points $P(-1, 2, 1)$, $Q(3, 1, 4)$, and $R(-2, 3, 5)$ lie on a plane.
   a. Using $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ as direction vectors and the point $R(-2, 3, 5)$, determine the Cartesian equation of this plane.
   b. Using $\overrightarrow{QP}$ and $\overrightarrow{PR}$ as direction vectors and the point $P(-1, 2, 1)$, determine the Cartesian equation of this plane.
   c. Explain why the two equations must be the same.
7. Determine the Cartesian equation of the plane that contains the points $A(-2, 3, 1)$, $B(3, 4, 5)$, and $C(1, 1, 0)$.
8. The line with vector equation $\vec{r} = (2, 0, 1) + s(-4, 5, 5)$, $s \in \mathbf{R}$, lies on the plane π, as does the point $P(1, 3, 0)$. Determine the Cartesian equation of π.
9. Determine unit vectors that are normal to each of the following planes:
   a. $2x + 2y - z - 1 = 0$
   b. $4x - 3y + z - 3 = 0$
   c. $3x - 4y + 12z - 1 = 0$
10. A plane contains the point $A(2, 2, -1)$ and the line $\vec{r} = (1, 1, 5) + s(2, 1, 3)$, $s \in \mathbf{R}$. Determine the Cartesian equation of this plane.

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11. Determine the Cartesian equation of the plane containing the point $(-1, 1, 0)$ and perpendicular to the line joining the points $(1, 2, 1)$ and $(3, -2, 0)$.

C

12. a. Explain the process you would use to determine the angle formed between two intersecting planes.
    b. Determine the angle between the planes $x - z + 7 = 0$ and $2x + y - z + 8 = 0$.
13. a. Determine the angle between the planes $x + 2y - 3z - 4 = 0$ and $x + 2y - 1 = 0$.
    b. Determine the Cartesian equation of the plane that passes through the point $P(1, 2, 1)$ and is perpendicular to the line $\frac{x-3}{-2} = \frac{y+1}{3} = \frac{z+4}{1}$.
14. a. What is the value of k that makes the planes $4x + ky - 2z + 1 = 0$ and $2x + 4y - z + 4 = 0$ parallel?
    b. What is the value of k that makes these two planes perpendicular?
    c. Can these two planes ever be coincident? Explain.

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15. Determine the Cartesian equation of the plane that passes through the points $(1, 4, 5)$ and $(3, 2, 1)$ and is perpendicular to the plane $2x - y + z - 1 = 0$.

PART C

16. Determine an equation of the plane that is perpendicular to the plane $x + 2y + 4 = 0$, contains the origin, and has a normal that makes an angle of $30°$ with the z-axis.
17. Determine the equation of the plane that lies between the points $(-1, 2, 4)$ and $(3, 1, -4)$ and is equidistant from them.