MCV4U Practice Exam: Vector Component

Part A: Multiple Choice

For questions 1 to 12, select the best answer.

1. Which is *not* an example of a vector?
A force **B** displacement
C speed **D** velocity

2. Which statement is *always* true?
A Parallel vectors have the same direction.
B Equivalent vectors have the same magnitude.
C Vectors are subtracted by adding the opposite.
D The resultant of two opposite vectors is the zero vector.

3. Given vectors $\vec{a}$ and $\vec{b}$ and scalar k, which is meaningless?
A $k\vec{a}$ **B** $\vec{a} \times b$
C $\vec{a} \cdot \vec{b}$ **D** $\vec{a}\ \vec{b}$

4. In three space, which is the definition of skew lines?
A Lines that intersect in a point.
B Non-parallel, non-intersecting lines.
C Lines that are perpendicular.
D Lines that are parallel.

5. Which vector equation represents a line through A(4, 3, 1) and B(−2, 1, 0)?
A $[x, y, z] = [4, 3, 1] + t[-2, 1, 0]$
B $[x, y, z] = [4, 3, 1] + t[2, 4, 1]$
C $[x, y, z] = [-2, 1, 0] + t[-6, -2, 1]$
D $[x, y, z] = [4, 3, 1] + t[6, 2, 1]$

6. Which expression is equivalent to $2(3\vec{i} - \vec{j} + \vec{k}) - (\vec{i} + 2\vec{k})$?
A [5, 2, 0] **B** $[5\vec{i} - 2\vec{j}]$
C [5, 2, 4] **D** $5\vec{i} - 2\vec{j}$

7. Which statement is *not* true?
A A line in two-space can be represented by a vector equation.
B A line in three-space can be represented by a scalar equation.
C A plane in three-space can be represented by a scalar equation.
D A plane in three-space can be represented by a vector equation.

8. Which scalar equation represents the same line as $[x, y] = [2, -2] + t[3, -1]$?
A $3x - y - 8 = 0$ **B** $x + 3y + 4 = 0$
C $3x + y - 4 = 0$ **D** $x - 3y + 8 = 0$

9. Which expression is meaningless?
A $\vec{a} \times \vec{b} \times \vec{c}$ **C** $\vec{a} \times \vec{b} \cdot \vec{c}$
B $\vec{a} \cdot \vec{b} \cdot \vec{c}$ **D** $(\vec{a} \cdot \vec{b}) \times \vec{c}$

10. Which statement is *not* correct?
A $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
B $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
C $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
D $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

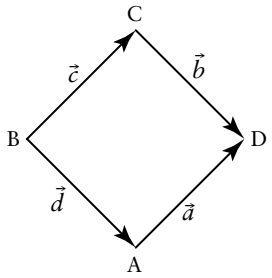
11. Which expression represents a unit vector in the same direction as [1, 2, −1]?
A [1, 1, 1] **B** $\frac{1}{\sqrt{6}}[1, 2, -1]$
C [1, 0, 0] **D** $\frac{1}{2}[1, 2, -1]$

12. Which statement best describes π_1 and π_2?
π_1: $2x - y + 3z - 4 = 0$
π_2: $4x - 2y + 6z - 7 = 0$
A π_1 and π_2 are parallel.
B π_1 and π_2 intersect in a single point.
C π_1 and π_2 are parallel and coincident.
D π_1 and π_2 are parallel and distinct.

Part B: Extended Response

Show all the steps of each solution.

13. Consider this diagram.



a) Name a vector that is equivalent to $\vec{a} - \vec{b}$.

b) Name a vector that is equivalent to $-\vec{b} - \vec{a}$.

14. The vertices of a triangle are P(−2, 3, 4), Q(3, −1, 1), and R(1, −2, −1).

a) Verify that △PQR is a right triangle.

b) Determine the area of △PQR.

c) Determine the coordinates of S(x, y, z) such that PQRS is a rectangle.

15. An airplane is headed N25°E with a constant velocity of 880 km/h. The plane encounters a wind blowing from S75°W at 65 km/h. Determine the resultant velocity of the plane.

16. A crate with mass 20 kg is suspended from a crane by two chains that make angles of 50° and 35° to the horizontal. Determine the tension in each chain.

17. Consider the vectors $\vec{u} = [-5, 1, -1]$ and $\vec{v} = [2, 4, -3]$.

a) Determine $\text{proj}_{\vec{u}}\,\vec{v}$.

b) Determine $\left|\text{proj}_{\vec{u}}\,\vec{v}\right|$.

18. A force $\vec{F} = [200, 600, 400]$, measured in newtons, acts on an object. The displacement of the object, in metres, is defined by $\vec{d} = [2, 1, 10]$.

a) Determine the work done in the direction of travel.

b) Determine the work done against gravity, which is a force in the direction of the negative z-axis.

19. Determine the equation of a plane that contains the line $[x, y, z] = [1, -2, 3] + t[4, 3, -5]$ and is parallel to the line $[x, y, z] = [1, 0, 9] + t[3, -2, 8]$.

20. Determine the intersection of the planes.
π_1: $3x - y + 4z - 1 = 0$
π_2: $x + 2y + z + 7 = 0$

21. Determine the intersection of these planes. Describe the solution geometrically.
π_1: $x + 3y + 2z - 5 = 0$
π_2: $2x - y - 4z - 4 = 0$
π_3: $4x - 3y + z + 3 = 0$