## Communicate Your Understanding

C1 Suzanne is unclear as to why the graphs of $y = x^2$ and $x = y^2$ are different, and why one is a function and the other is not. How would you help Suzanne?

C2 Is it possible to determine if a relation is a function if you are only given the domain and range in set notation? Explain your reasoning.

C3 Sagar missed the class on restrictions and has asked you for help. Lead him through the steps needed to find the domain and range of the function $y = \dfrac{-4}{2x + 1}$.

A Practise

For help with questions 1 and 2, refer to Example 1.

1. Which graphs represent functions? Justify your answer.

a)

b)

c)

d)
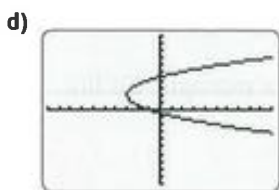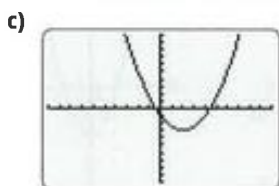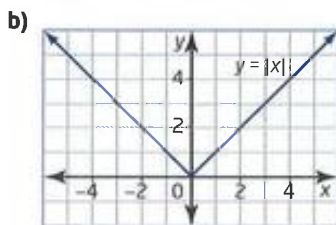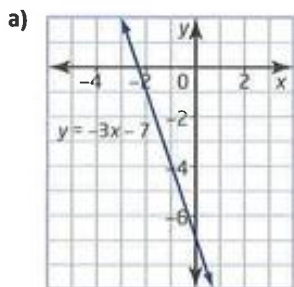
2. Is each relation a function? Explain. Sketch a graph of each.

a) $y = x - 5$

b) $x = y^2 - 3$

c) $y = 2(x - 1)^2 - 2$

d) $x^2 + y^2 = 4$

For help with questions 3 and 4, refer to Example 2.

3. State the domain and the range of each relation. Is each relation a function? Justify your answer.

a) $\{(5, 5), (6, 6), (7, 7), (8, 8), (9, 9)\}$

b) $\{(3, -1), (4, -1), (5, -1), (6, -1)\}$

c) $\{(1, 6), (1, -14), (1, 11), (1, -8), (1, 0)\}$

d) $\{(1, 5), (4, 11), (3, 9), (5, 1), (11, 4)\}$

e) $\{(3, 2), (2, 1), (1, 0), (2, -1), (3, -2)\}$

4. The domain and range of some relations are given. Each relation consists of five points. Is each a function? Explain.
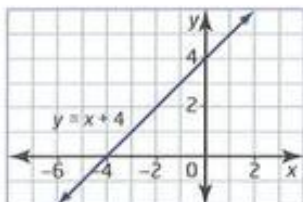
a) domain $\{1, 2, 3, 4, 5\}$, range $\{4\}$

b) domain $\{-3, -1, 1, 3, 5\}$,
range $\{2, 4, 6, 8, 10\}$

c) domain $\{2, 3, 6\}$,
range $\{-4, 6, 7, 11, 15\}$

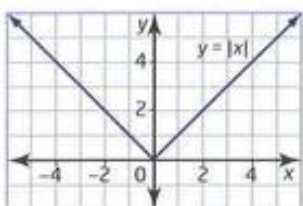**d)** domain $\{-2\}$, range $\{9, 10, 11, 12, 13\}$

For help with questions 5 and 6, refer to Example 3.

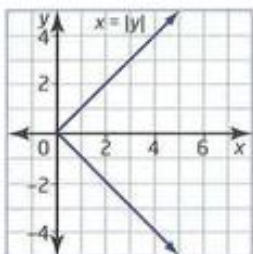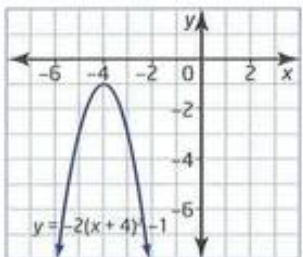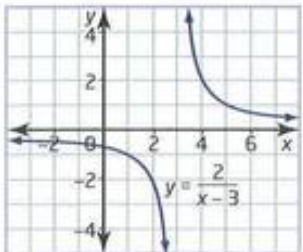**5.** State the domain and the range of each relation.

a)

b)

c)

d)

e)

6. Determine the domain and the range of each relation. Use a graph to help you if necessary.

a) $y = -x + 3$
b) $y = (x + 1)^2 - 4$
c) $y = -3x^2 + 1$
d) $x^2 + y^2 = 9$
e) $y = \dfrac{1}{x + 3}$
f) $y = \sqrt{2x + 1}$

7. For each given domain and range, draw one relation that is a function and one that is not. Use the same set of axes for each part.

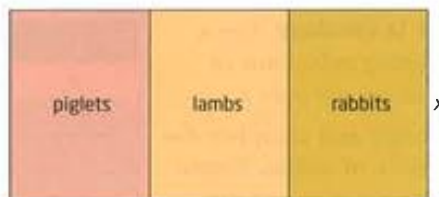a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$
b) domain $\{x \in \mathbb{R}, x \geq 4\}$, range $\{y \in \mathbb{R}\}$
c) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq -1\}$
d) domain $\{x \in \mathbb{R}, x \leq 2\}$, range $\{x \in \mathbb{R}, y \geq -2\}$

For help with questions 8 and 9, refer to Example 4.

8. Soula has 90 m of fencing to enclose an area in a petting zoo with two dividers to separate three types of young animals. The three pens are to have the same area.

a) Express the area function for the three pens in terms of x.
b) Determine the domain and the range for the area function.

9. Is each relation a function? Justify your answer. If the relation is a function, state the independent variable and the dependent variable.

a) The amount of money taken in for the fundraiser is related to the number of raffle tickets a hockey team sells.
b) The age of students is related to their grade level.
c) The time it takes Jung Yoo to walk to school is related to the speed at which he walks.

10. A rectangular part of a parking lot is to be fenced off to allow some repairs to be done. The workers have fourteen 3-m sections of pre-assembled fencing to use. They want to create the greatest possible area in which to work.

Reasoning and Proving
Representing — *Selecting Tools*
Problem Solving
Connecting — *Reflecting*
Communicating

a) How can the fencing be used to create as large an enclosed area as possible?

b) Show why this produces the greatest area using the given fencing sections, but does not create the greatest area that can be enclosed with 42 m of fencing.

11. Determine the range of each relation for the domain {1, 2, 3, 4, 5}.

a) $y = 6x - 6$ **b)** $y = x^2 - 4$

c) $y = 3$ **d)** $y = 2(x - 1)^2 - 1$

e) $y = \dfrac{1}{x + 2}$ **f)** $x^2 + y^2 = 25$

12. Use Technology Use a graphing calculator or graphing software.

a) Copy and complete the table of values. Create a scatter plot of the resulting data using a graphing calculator.

$x = y^2 - 3$	y
6	-3
	-2
	-1
	0
	1
	2
	3

b) Enter the equations

$y = \sqrt{x + 3}$ and

$y = -\sqrt{x + 3}$ and display their graphs.

c) Explain the result of the display of the data and the equations.

d) Explain how this illustrates that the equation $x = y^2 - 3$ defines a relation that is not a function.

> **Technology Tip**
>
> Refer to the Technology Appendix, pages 496 to 516, if you need help with plotting data or graphing equations.

13. It is said that you cannot be in two places at once. Explain what this statement means in terms of relations and functions.

14. Describe the graph of a relation that has

a) one entry in the domain and one entry in the range

b) one entry in the domain and many entries in the range

c) many entries in the domain and one entry in the range

15. Sketch a relation with the following properties.

a) It is a function with domain all the real numbers and range all real numbers less than or equal to 5.

b) It is not a function and has domain and range from -3 to 3.

16. A car salesperson is paid according to two different relations based on sales for the week. In both relations, s represents sales and P represents the amount paid, both in dollars.

Reasoning and Proving
Representing — *Selecting Tools*
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For sales of less than $100 000,
$P = 0.002s + 400$.
For sales of $100 000 and over,
$P = 0.0025s + 400$.

a) State the domain and range for each relation.

b) Does each relation define a function? Justify your answer.

c) Graph the two relations on the same set of axes.

d) Connect what happens on the graph at $s = 100\ 000$ to its meaning for the salesperson.
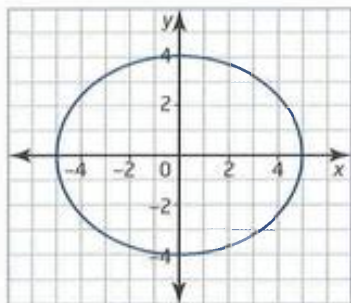
C Extend

17. Is it possible for two different functions to have the same domain and range? Explain, giving examples.

18. State the domain and the range for the two relations shown. Is each a function?

a)
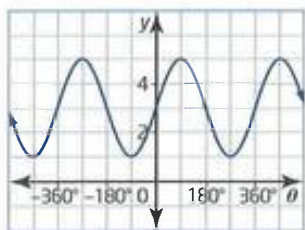
b)

19. Math Contest What is the domain of the function $y = \dfrac{\sqrt{x-3}}{\sqrt{5-x}}$?

20. Math Contest Frank bought supplies for school. In the first store, he spent half his money plus $10. In the second store, he spent half of what he had left plus $10. In the third store, he spent 80% of what he had left. He came home with $5. How much did he start out with?

21. Math Contest Find the number of factors of 2520.

22. Math Contest For what values of x is $\sqrt{x+2} > x$?

Career Connection

Khaldun completed a 4-year degree in mineral engineering at the University of Toronto. He works in northern Canada for an international diamond-mining company. In his job as a mining engineer, Khaldun uses his knowledge of mathematics, physics, geology, and environmental science to evaluate the feasibility of a new mine location. Whether the mine is excavated will be a function of the value of the diamond deposit, accessibility, and safety factors. Since mining a site costs millions of dollars, the analysis stage is crucial. Khaldun examines rock samples and the site itself before carefully estimating the value of the underground deposit. The diamonds will be mined only if the profits outweigh the many costs.