

1.3 Factoring

$$3(x+2) \xrightleftharpoons[\text{Factor}]{\text{Expand/Multiply}} 3x+6$$

To factor is to write an algebraic expression as a **product** of two or more other algebraic expressions .

Why factor? To arrive at equivalent expressions which are presented in simpler terms which allows us to:

- Solve equations
- Graph relations

In grade 10 you learned how to:

- Common Factor
- Factor by Grouping
- Factor Simple Trinomials
- Factor Complex Trinomials
- Factor a Difference of Squares
- Factor a Perfect Square Trinomial

Common Factoring



Always your first and last step.



WHEN?

2 or more terms

HOW?

- Take out the greatest common factor.
- Divide the expression by the GCF to find the other factor.

a) $2mn - 4mnt$
 $= 2mn(1 - 2t)$

b) $6t^5 - 9t^2$
 $= 3t^2(2t^3 - 3)$

c) $3x^4 - 6x^3 + 9x$
 $= 3x(x^3 - 2x^2 + 3)$

d) $4x(a-b) - 3(a-b)$
 $= (a-b)(4x - 3)$

Factor by Grouping

WHEN?

An even # of terms: 4, 6, 8, etc...

HOW?

- Group terms to form pairs.
- Factor the pairs by finding common factors.
- Factor out the shared common binomial factor.

$$\begin{aligned} \text{a) } & 3x(m-5) + 2(5-m) \\ &= 3x(m-5) + 2(-1)(m-5) \\ &= (m-5)(3x-2) \end{aligned}$$



The terms $m-5$ and $5-m$ are opposites. This means that one divided by the other is -1 .

$$\begin{aligned} \text{b) } & x(y-2) - 4(2-y) \\ &= x(y-2) + 4(y-2) \\ &= (y-2)(x+4) \end{aligned}$$

$$\begin{aligned} \text{c) } & mx + 2y + my + 2x \\ &= mx + my + 2x + 2y \\ &= m(x+y) + 2(x+y) \\ &= (x+y)(m+2) \end{aligned}$$

$$\begin{aligned} \text{d) } & 22vx - 6vy + 11wx - 3wy \\ &= -6vy - 3wy + 11wx + 22vx \\ &= -3y(2v+w) + 11x(w+2v) \\ &= (2v+w)(-3y+11x) \end{aligned}$$

$$\begin{aligned} \text{e) } & y^2 + 1 - y^3 - y \\ &= 1(y^2+1) - y(y^2+1) \\ &= (y^2+1)(1-y) \end{aligned}$$

$$\begin{aligned} \text{f) } & 16x^5 + 8x^4 - 6x^3 - 3x^2 + 4x + 2 \\ &= 8x^4(2x+1) - 3x^2(2x+1) + 2(2x+1) \\ &= (2x+1)(8x^4 - 3x^2 + 2) \end{aligned}$$

Simple Trinomials

WHEN?

3 terms

 $ax^2 + bx + c$ where $a = 1$

a) $x^2 - 9x + 14$

$= (x-2)(x-7)$

M 14

A -9

N -2, -7

HOW?

$(x + n_1)(x + n_2)$

M = ac

A = b

N = n_1, n_2

b) $5x^2 + 15x - 140$

$= 5(x^2 + 3x - 28)$
 $= 5(x+7)(x-4)$

M -28

A 3

N 7, -4

c) $a^2 + 8ab + 15b^2$

$= (a+5b)(a+3b)$

M $15b^2$

A $8b$

N $5b, 3b$

d) $x^4 + 2x^2b - 24b^2$

$= (x^2 + 6b)(x^2 - 4b)$

M $-24b^2$

A $+2b$

N $6b, -4b$

Difference of Squares

WHEN?

2 terms

2 perfect squares separated
by a subtraction: $a^2 - b^2$

a) $49x^2 - 16y^2$

$$= (7x - 4y)(7x + 4y)$$

c) $a^2 - \frac{1}{9}$

$$= \left(a + \frac{1}{3}\right)\left(a - \frac{1}{3}\right)$$

e) $(3x - 2)^2 - (5x + 1)^2$

$$= (3x - 2 + 5x + 1)(3x - 2 - (5x + 1))$$

$$= (8x - 1)(-2x - 3)$$

HOW?

$$a^2 - b^2 = (a - b)(a + b)$$

conjugates

b) $3x^2 - 12$

$$= 3(x^2 - 4)$$

$$= 3(x - 2)(x + 2)$$

d) $81 - m^{12}$

$$= (9 + m^6)(9 - m^6)$$

$$\begin{aligned} 81x^2 - 9y^2 \\ &= (9x)^2 - (3y)^2 \\ &= (9x + 3y)(9x - 3y) \end{aligned}$$

Homework Handout

Complex Trinomials

WHEN?

3 terms

$$ax^2 + bx + c \text{ where } a \neq 1$$

HOW?

$$(a_1x + f_1)(a_2x + f_2)$$

$$M = ac$$

$$A = b$$

$$N = n_1, n_2$$

1. Use a , n_1 and n_2
to find the factors.

$$\frac{a}{n_1}, \frac{a}{n_2}$$

2. Reduce.

$$\frac{a_1}{f_1}, \frac{a_2}{f_2}$$

OR

Decompose the middle term
using n_1 , n_2 and factor by
grouping.

a) $10x^2 - 11x - 6$

$$\begin{aligned} &= 10x^2 - 15x + 4x - 6 \\ &= 5x(2x - 3) + 2(2x - 3) \\ &= (2x - 3)(5x + 2) \end{aligned}$$

$$M - 60$$

$$A - 11$$

$$N - 15, 4$$

a) $10x^2 - 11x - 6$

$$= (2x - 3)(5x + 2)$$

$$M - 60$$

$$A - 11$$

$$N - 15$$

$$\frac{4}{10}$$

$$-\frac{3}{2}$$

$$\frac{2}{5}$$

b) $14x^2 + 31xy - 10y^2$

$$M - 140$$

$$= (2x + 5y)(7x - 2y)$$

$$A 31$$

$$N$$

$$\frac{35}{14} \quad -\frac{4}{14}$$

c) $18a^2b + 3ab - 6b$

$$= 3b(6a^2 + a - 2)$$

$$= 3b(3a + 2)(2a - 1)$$

$$M - 12$$

$$A 1$$

$$N \quad \frac{4}{6} \quad -\frac{3}{6}$$

$$\frac{2}{3} \quad -\frac{1}{2}$$

d) $3x^4 - 25x^2 - 18$

$$= (x^2 - 9)(3x^2 + 2)$$

$$= (x + 3)(x - 3)(3x^2 + 2)$$

$$\frac{5}{2} \quad -\frac{2}{7}$$

$$M - 54$$

$$A - 25$$

$$N \quad -\frac{27}{3} \quad \frac{2}{3}$$

$$-\frac{9}{1}$$

Perfect Square Trinomials

WHEN?

3 terms

$$ax^2 + bx + c$$

where a & c are perfect squares and b is twice the product of their square roots.

HOW?

$$(\sqrt{a}x \pm \sqrt{c})^2$$

same sign as b

a) $m^2 + 10m + 25$

$$= (m+5)^2$$

b) $2x^2 - 24x + 72$

$$= 2(x^2 - 12x + 36)$$

$$= 2(x-6)^2$$

c) $16a^2 + 24a + 9$

$$= (4a+3)^2$$

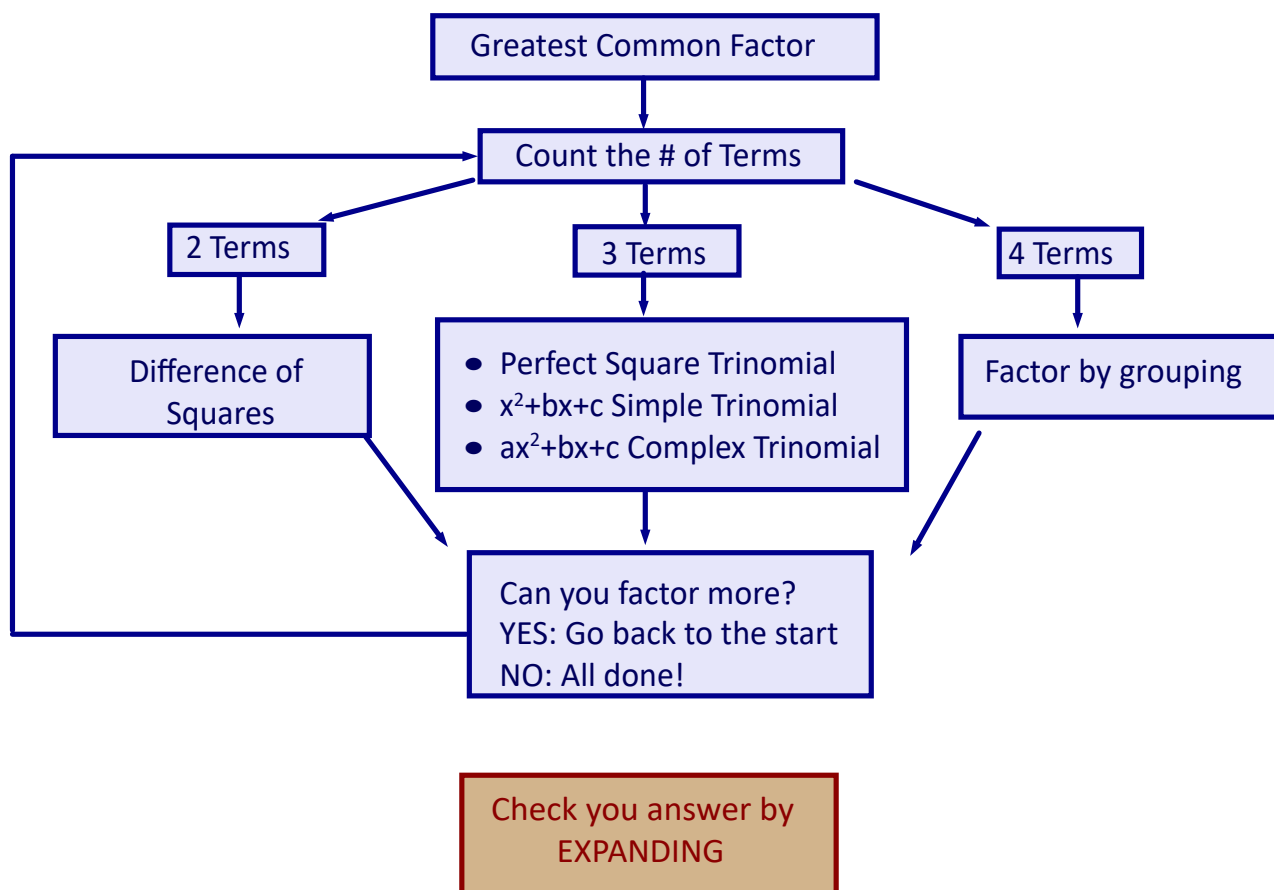
d) $x^4 - 8x^2 + 16$

$$= (x^2 - 4)^2$$

$$= [(x-2)(x+2)]^2$$

$$= (x-2)^2 (x+2)^2$$

Factoring Flowchart



HOMework

Handout 1.3

The image shows a handwritten solution for factoring the quadratic expression $2x^2 + 9x + 10$ on a chalkboard background. The expression is written in yellow at the top, with the first and last terms underlined. Below it, the numbers 2 and 5 are written in red above a 2x2 grid. The grid contains the terms $2x^2$, $5x$, $4x$, and 10 in orange. To the left of the grid, the numbers 1 and 2 are written in red. To the right, the number 20 is written in orange with an arrow pointing to the grid, and a list of factors (1, 2, 4, 5, 10, 20) is written in yellow, with 4 and 5 circled. At the bottom, the factored form $(x+2)(2x+5)$ is written in blue.

$$2x^2 + 9x + 10$$

	$2x$	5
x	$2x^2$	$5x$
2	$4x$	10

$(x+2)(2x+5)$

Factors of 20: 1, 2, 4, 5, 10, 20. (4 and 5 are circled)