1.3 Factoring

$$3(x+2) \underset{\text{Factor}}{\overset{\text{Expand/Multiply}}{\rightleftarrows}} 3x+6$$

To factor is to write an algebraic expression as a product of two or more other algebraic expressions .

Why factor? To arrive at equivalent expressions which are presented in simpler terms which allows us to:

- Solve equations
- Graph relations

In grade 10 you learned how to:

- Common Factor
- Factor by Grouping
- Factor Simple Trinomials
- Factor Complex Trinomials
- Factor a Difference of Squares
- Factor a Perfect Square Trinomial

Common Factoring

Always your first and last step.

2 or more terms

- Take out the greatest common factor.
- Divide the expression by the GCF to find the other factor.

a) $2mn-4mnt$

$=2mn(1-2t)$

b) $6t^5-9t^2$

$=3t^2(2t^3-3)$

c) $3x^4-6x^3+9x$

$=3x(x^3-2x^2+3)$

d) $4x(a-b)-3(a-b)$

$=(a-b)(4x-3)$

Factor by Grouping

An even # of terms: 4, 6, 8, etc...

- Group terms to form pairs.
- Factor the pairs by finding common factors.
- Factor out the shared common binomial factor.

a) $3x(m-5)+2(5-m)$

$= 3x(m-5)+2(-1)(m-5)$

$= (m-5)(3x-2)$

! The terms $m - 5$ and $5 - m$ are opposites. This means that one divided by the other is -1.

b) $x(y-2)-4(2-y)$

$= x(y-2)+4(y-2)$

$= (y-2)(x+4)$

c) $mx+2y+my+2x$

$= mx+my+2x+2y$

$= m(x+y)+2(x+y)$

$= (x+y)(m+2)$

d) $22vx-6vy+11wx-3wy$

$= -6vy-3wy+11wx+22vx$

$= -3y(2v+w)+11x(w+2v)$

$= (2v+w)(-3y+11x)$

e) y^2+1-y^3-y

$= 1(y^2+1)-y(y^2+1)$

$= (y^2+1)(1-y)$

f) $16x^5+8x^4-6x^3-3x^2+4x+2$

$= 8x^4(2x+1)-3x^2(2x+1)+2(2x+1)$

$= (2x+1)(8x^4-3x^2+2)$

Simple Trinomials

WHEN?

3 terms

$ax^2 + bx + c$ where $a = 1$

HOW?

$(x + n_1)(x + n_2)$

$M = ac$

$A = b$

$N = n_1, n_2$

a) $x^2 - 9x + 14$

$= (x-2)(x-7)$

M 14

A −9

N −2, −7

b) $5x^2 + 15x - 140$

$= 5(x^2 + 3x - 28)$

$= 5(x+7)(x-4)$

M −28

A 3

N 7, −4

c) $a^2 + 8ab + 15b^2$

$= (a+5b)(a+3b)$

M $15b^2$

A $8b$

N $5b, 3b$

d) $x^4 + 2x^2b - 24b^2$

$= (x^2 + 6b)(x^2 - 4b)$

M $-24b^2$

A $+2b$

N $6b, -4b$

Difference of Squares

2 terms

2 perfect squares separated by a subtraction: $a^2 - b^2$

$a^2 - b^2 = (a - b)(a + b)$

conjugates

a) $49x^2 - 16y^2$

$= (7x - 4y)(7x + 4y)$

b) $3x^2 - 12$

$= 3(x^2 - 4)$

$= 3(x - 2)(x + 2)$

c) $a^2 - \frac{1}{9}$

$= (a + \frac{1}{3})(a - \frac{1}{3})$

d) $81 - m^{12}$

$= (9 + m^6)(9 - m^6)$

e) $(3x-2)^2 - (5x+1)^2$

$= (3x - 2 + 5x + 1)(3x - 2 - (5x + 1))$

$= (8x - 1)(-2x - 3)$

$81x^2 - 9y^2$

$= (9x)^2 - (3y)^2$

$= (9x + 3y)(9x - 3y)$

Homework Handout

Complex Trinomials

WHEN?

3 terms

$ax^2 + bx + c$ where $a \neq 1$

HOW?

$(a_1x + f_1)(a_2x + f_2)$

M = ac
A = b
N = n_1, n_2

1. Use a, n_1 and n_2 to find the factors.

$$\frac{a}{n_1}, \frac{a}{n_2}$$

2. Reduce.

$$\frac{a_1}{f_1}, \frac{a_2}{f_2}$$

OR

Decompose the middle term using n_1, n_2 and factor by grouping.

a) $10x^2 - 11x - 6$

$= 10x^2 - 15x + 4x - 6$
$= 5x(2x-3) + 2(2x-3)$
$= (2x-3)(5x+2)$

M -60
A -11
N $-15, 4$

a) $10x^2 - 11x - 6$

$= (2x-3)\ (5x+2)$

M -60
A -11
N $-15 \quad 4$

$\frac{-15}{10} \quad \frac{4}{10}$

$-\frac{3}{2} \quad \frac{2}{5}$

b) $14x^2 + 31xy - 10y^2$

$= (2x+5y)(7x-2y)$

M -140
A 31
N

$\frac{35}{14} \quad \frac{-4}{14}$

$\frac{5}{2} \quad \frac{-2}{7}$

c) $18a^2b + 3ab - 6b$

$= 3b(6a^2 + 1a - 2)$

$= 3b(3a+2)(2a-1)$

M -12
A 1
N $4 \quad -3$

$\frac{4}{6} \quad \frac{-3}{6}$

$\frac{2}{3} \quad -\frac{1}{2}$

d) $3x^4 - 25x^2 - 18$

$= (x^2-9)(3x^2+2)$

$= (x+3)(x-3)(3x^2+2)$

M -54
A -25
N $-27 \quad 2$

$\frac{-27}{3} \quad \frac{2}{3}$

$\frac{-9}{1}$

Perfect Square Trinomials

WHEN?

3 terms

$ax^2 + bx + c$

where a & c are perfect squares and
b is twice the product of their square roots.

HOW?

$\left(\sqrt{a}x \pm \sqrt{c}\right)^2$

same sign as b

a) $m^2 + 10m + 25$

$= (m+5)^2$

b) $2x^2 - 24x + 72$

$= 2(x^2 - 12x + 36)$

$= 2(x-6)^2$

c) $16a^2 + 24a + 9$

$= (4a+3)^2$

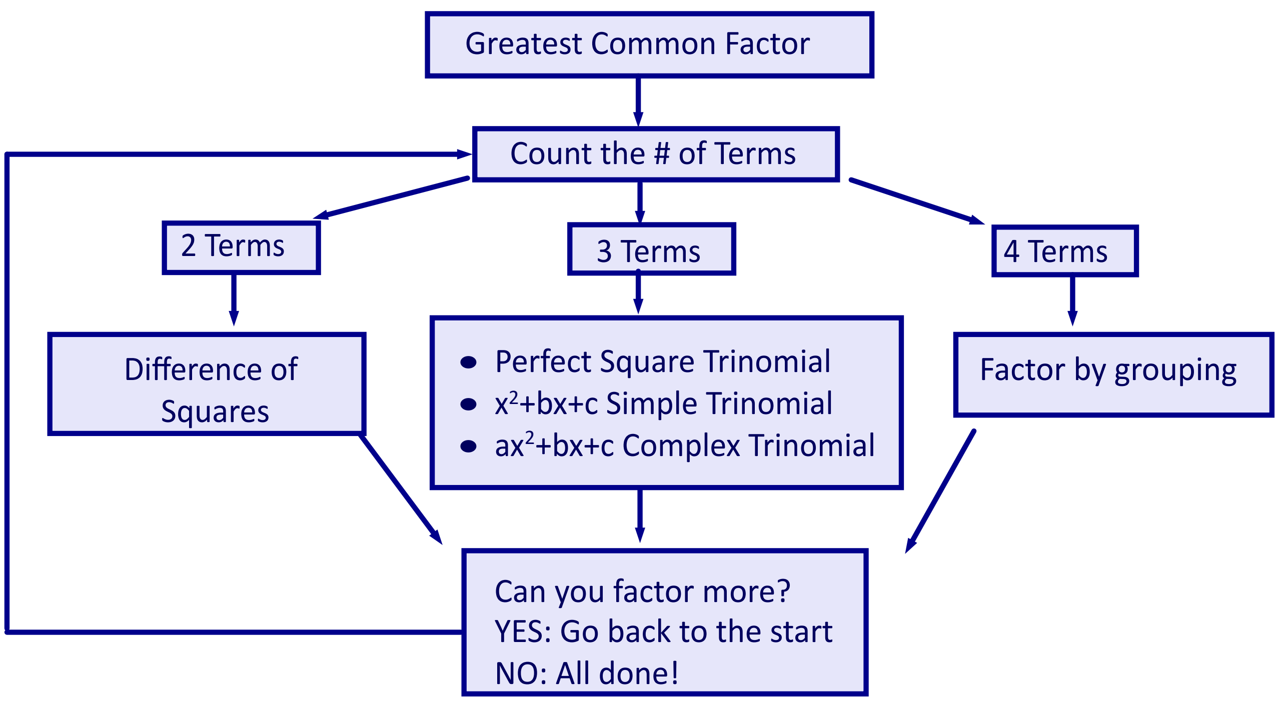
d) $x^4 - 8x^2 + 16$

$= (x^2 - 4)^2$

$= [(x-2)(x+2)]^2$

$= (x-2)^2(x+2)^2$

Factoring Flowchart



Check you answer by EXPANDING

HOMEWORK
Handout 1.3