1.3 Factoring

Expand/Multiply

3(x+2) ⟶ 3x+6

3(x+2) ⟵ 3x+6

Factor

To factor is to write an algebraic expression as a product of two or more other algebraic expressions .

Why factor? To arrive at equivalent expressions which are presented in simpler terms which allows us to:

- Solve equations
- Graph relations

In grade 10 you learned how to:

- Common Factor
- Factor by Grouping
- Factor Simple Trinomials
- Factor Complex Trinomials
- Factor a Difference of Squares
- Factor a Perfect Square Trinomial

Common Factoring

Always your first and last step.

2 or more terms

- Take out the greatest common factor.
- Divide the expression by the GCF to find the other factor.

a) $2mn - 4mnt$

$= 2mn(1-2t)$

b) $6t^5 - 9t^2$

$= 3t^2(2t^3-3)$

c) $3x^4 - 6x^3 + 9x$

$= 3x(x^3-2x^2+3)$

d) $4x(a-b) - 3(a-b)$

$= (a-b)(4x-3)$

Factor by Grouping

An even # of terms: 4, 6, 8, etc...

- Group terms to form pairs.
- Factor the pairs by finding common factors.
- Factor out the shared common binomial factor.

The terms *m - 5* and *5 - m* are opposites. This means that one divided by the other is *-1*.

a) $3x(m-5)+2(5-m)$

$= 3x(m-5)+2(-1)(m-5)$

$= (m-5)(3x-2)$

b) $x(y-2)-4(2-y)$

$= x(y-2)+4(y-2)$

$= (y-2)(x+4)$

c) $mx+2y+my+2x$

$= mx+my+2x+2y$

$= m(x+y)+2(x+y)$

$= (x+y)(m+2)$

d) $22vx-6vy+11wx-3wy$

$= -6vy-3wy+11wx+22vx$

$= -3y(2v+w)+11x(w+2v)$

$= (2v+w)(-3y+11x)$

e) y^2+1-y^3-y

$= 1(y^2+1)-y(y^2+1)$

$= (y^2+1)(1-y)$

f) $16x^5+8x^4-6x^3-3x^2+4x+2$

$= 8x^4(2x+1)-3x^2(2x+1)+2(2x+1)$

$= (2x+1)(8x^4-3x^2+2)$

Simple Trinomials

3 terms

$ax^2 + bx + c$ where $a = 1$

$(x + n_1)(x + n_2)$

M = ac
A = b
N = n_1, n_2

a) $x^2 - 9x + 14$

$= (x-2)(x-7)$

M 14
A −9
N −2, −7

b) $5x^2 + 15x - 140$

$= 5(x^2 + 3x - 28)$

$= 5(x+7)(x-4)$

M −28
A 3
N 7, −4

c) $a^2 + 8ab + 15b^2$

$= (a+5b)(a+3b)$

M $15b^2$
A 8b
N 5b, 3b

d) $x^4 + 2x^2b - 24b^2$

$= (x^2 + 6b)(x^2 - 4b)$

M $-24b^2$
A +2b
N 6b, −4b

Difference of Squares

2 terms

2 perfect squares separated by a subtraction: $a^2 - b^2$

$a^2 - b^2 = (a - b)(a + b)$

conjugates

a) $49x^2 - 16y^2$

$= (7x - 4y)(7x + 4y)$

b) $3x^2 - 12$

$= 3(x^2 - 4)$

$= 3(x - 2)(x + 2)$

c) $a^2 - \frac{1}{9}$

$= (a + \frac{1}{3})(a - \frac{1}{3})$

d) $81 - m^{12}$

$= (9 + m^6)(9 - m^6)$

e) $(3x-2)^2 - (5x+1)^2$

$= (3x - 2 + 5x + 1)(3x - 2 - (5x + 1))$

$= (8x - 1)(-2x - 3)$

$81x^2 - 9y^2$

$= (9x)^2 - (3y)^2$

$= (9x + 3y)(9x - 3y)$

Homework Handout

Complex Trinomials

3 terms

$ax^2 + bx + c$ where $a \neq 1$

$(a_1x + f_1)(a_2x + f_2)$

M = ac
A = b
N = n_1, n_2

1. Use a, n_1 and n_2 to find the factors.

$\frac{a}{n_1}, \frac{a}{n_2}$

2. Reduce.

$\frac{a_1}{f_1}, \frac{a_2}{f_2}$

OR

Decompose the middle term using n_1, n_2 and factor by grouping.

a) $10x^2 - 11x - 6$

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b) $14x^2 + 31xy - 10y^2$

c) $18a^2b + 3ab - 6b$

d) $3x^4 - 25x^2 - 18$

Perfect Square Trinomials

3 terms

$ax^2 + bx + c$

where a & c are perfect squares and
b is twice the product of their square roots.

$$\left(\sqrt{a}x \pm \sqrt{c}\right)^2$$

same sign as b

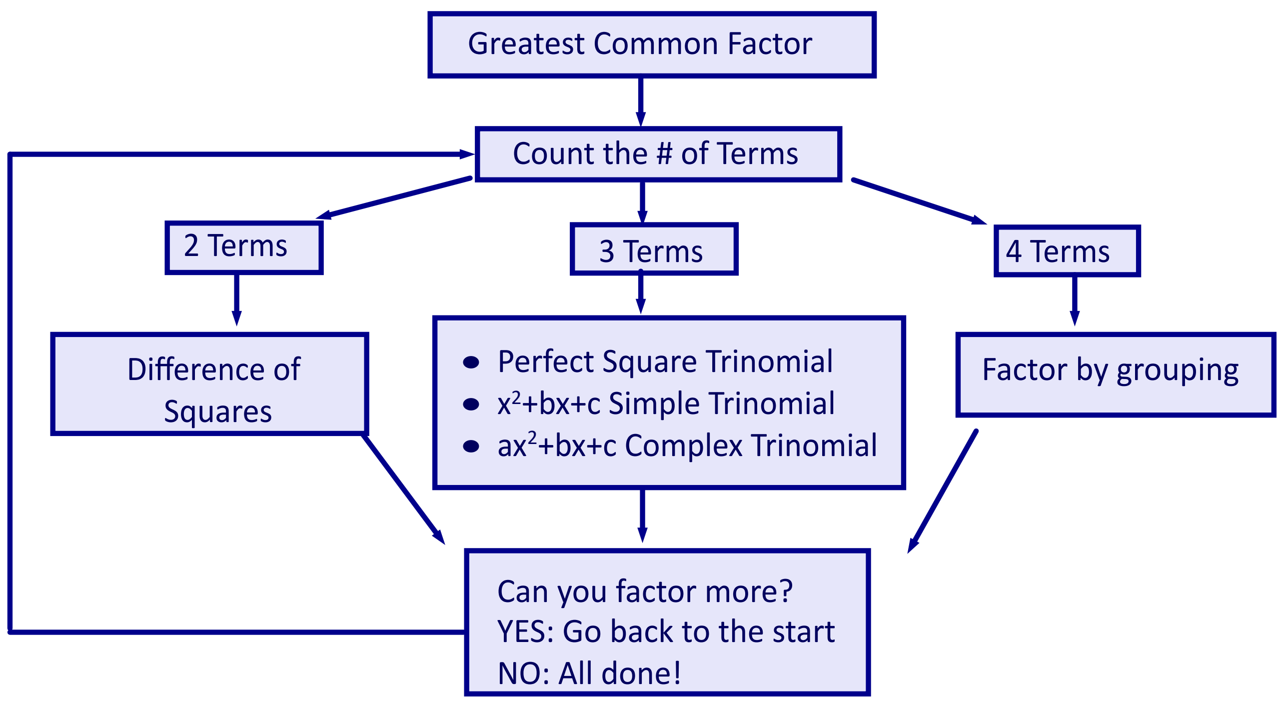
a) $m^2 + 10m + 25$

b) $2x^2 - 24x + 72$

c) $16a^2 + 24a + 9$

d) $x^4 - 8x^2 + 16$

Factoring Flowchart



Check you answer by EXPANDING

HOMEWORK
Handout 1.3