

1.6 Solve Quadratic Equations

Recall: Solving a quadratic equation means finding the value of the roots, zeros or x-intercepts. You are finding where the function, $f(x)$ is zero.

A. Solve by Factoring

Ex. 1 Solve each of the following:

a) $f(x) = (x-3)(x+4)$

$$0 = (x-3)(x+4)$$

$$\begin{array}{cc} x-3=0 & \downarrow \\ x=3 & x=3 \end{array} \quad \begin{array}{cc} & \downarrow \\ & x=-4 \end{array}$$

b) $f(x) = x^2 + 7x - 30$

$$= (x+10)(x-3)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x=-10 & x=3 \end{array}$$

How do you find zeros?

1. Set $f(x) = 0$.
2. Factor.
3. Set each factor = 0 and solve for x.

c) $f(x) = 4x^2 - 9$

$$= (2x+3)(2x-3)$$

$$\begin{array}{cc} 2x+3=0 & \downarrow \\ 2x=-3 & x=-\frac{3}{2} \\ x=-\frac{3}{2} & \end{array} \quad \begin{array}{cc} & \downarrow \\ & x=\frac{3}{2} \end{array}$$

d) $f(x) = 3x^2 + 12x$

$$= 3x(x+4)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x=0 & x=-4 \end{array}$$

e) Find the vertex of d)

$$\begin{array}{l} \text{axis} \\ x = \frac{0+(-4)}{2} \\ = -2 \end{array}$$

$$\begin{array}{l} f(-2) = 3(-2)(-2+4) \\ = -6(2) \\ = -12 \end{array}$$

$$\therefore \text{Vertex}(-2, -12)$$

B. Solve from Vertex Form

Ex. 1 Solve each of the following:

$f(x) = 2(x-3)^2 - 8$

$$8 = 2(x-3)^2$$

$$4 = (x-3)^2$$

$$\pm\sqrt{4} = x-3$$

$$3 \pm 2 = x$$

$$\begin{array}{l} x = 3 + 2 \\ = 5 \end{array}$$

$$\begin{array}{l} x = 3 - 2 \\ = 1 \end{array}$$

1. Set $y = 0$.
2. Isolate for x

C. Solve using the Quadratic Formula

Recall:

Requires Std form: $ax^2 + bx + c$

Exact answers only!!!

The quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex. 1 Solve. Give exact answers only.

$a = 3 \quad b = 4 \quad c = -2$

a) $3x^2 + 4x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{16 + 24}}{6}$$

$$= \frac{-4 \pm \sqrt{40}}{6}$$

$$= \frac{-4 \pm 2\sqrt{10}}{6}$$

$$= \frac{-2 \pm \sqrt{10}}{3}$$

$a = 5 \quad b = -3 \quad c = 2$

b) $5x^2 - 3x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{3 \pm \sqrt{(-3)^2 - 4(5)(2)}}{2(5)}$$

$$= \frac{3 \pm \sqrt{-31}}{10}$$

\therefore No solⁿ
 \therefore No zeros

We can determine the number of roots
 by looking under the radical sign

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

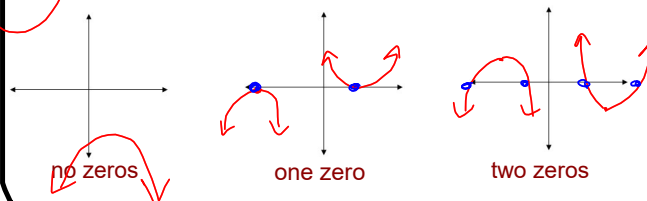
This is known as the

Discriminant

$b^2 - 4ac$

Quadratics can have no zeros, 1 zero, or 2 zeros.

Sketch an example of each scenario:



➡ If $b^2 - 4ac > 0$ then there is two real roots

➡ If $b^2 - 4ac = 0$ then there is one real root

➡ If $b^2 - 4ac < 0$ then there is no real roots

Ex. 2 For each quadratic equation, determine the number of roots.

a) $2x^2 - x + 5 = 0$

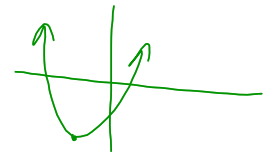
$$\begin{aligned} D &= b^2 - 4ac \\ &= (-1)^2 - 4(2)(5) \\ &= 1 - 40 \\ &= -39 \end{aligned}$$

$D < 0$
 \therefore No roots

b) $4(x + 1)^2 - 7 = 0$

$$V(-1, -7)$$

Opens UP!



\therefore 2 zeroes

c) $(x - 6)^2 = 0$

$$V(6, 0) \text{ (on the axis!)}$$

\therefore One zero

d) $2x^2 + 8x + 8 = 0$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 8^2 - 4(2)(8) \\ &= 64 - 64 \\ &= 0 \end{aligned}$$

\therefore One zero

Homework
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