Lesson 1.7A: Determining a Quadratic Equation Given Its Roots

Recall: Quadratics can be represented in a number of different forms:

Note: All forms give "a" (dir. of opening) aka stretch factor

Standard Form

$$f(x) = ax^2 + bx + c$$

What do each of these forms tell you about the quadratic function?

complete the square → Vertex Form

factor → Factored Form

partial factor → Partially Factored Form

Vertex Form	Factored Form	Partially Factored Form
$f(x) = a(x-h)^2 + k$	$f(x) = a(x-r)(x-s)$	$f(x) = ax(x-s) + t$
Vertex (h, k)	x-ints are r & s	Gives two symmetrical pts.
k is max/min	Axis of symm: $x = \frac{r+s}{2}$	$(0, t)$ & (s, t)
h is when it happens	Sub in to solve for y of vertex	Axis of symm: $x = \frac{0+s}{2}$

Ex. 1 Find the equation, in factored form, for a family of quadratic functions that has zeros at x = 4 and x = -3. Sketch three possible members of this family.

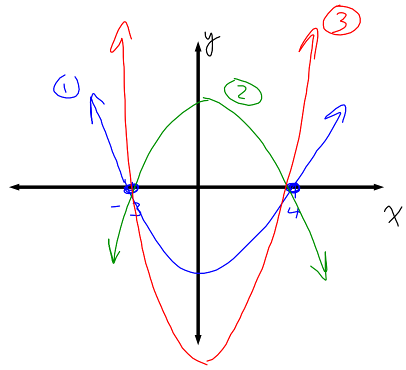
Factored form!

$y = a(x-r)(x-s)$

Sub in $x=4$ & $x=-3$

$y = a(x-4)(x-(-3))$

$= a(x-4)(x+3)$



① $y = (x-4)(x+3)$

② $y = -(x-4)(x+3)$

③ $y = 2(x-4)(x+3)$

Ex. 2 Algebraically determine the equation of the quadratic function, in standard form, having only one x-intercept, at x = 2 (double root), and containing the point (3,10).

- Start with a form that uses the data provided.
- Expand into standard form

$y = a(x-h)^2 + k$

Sub in $(2,0)$ for (h,k)

$y = a(x-2)^2$

Sub in $(3,10)$ to solve for a

$10 = a(3-2)^2$

$10 = a(1)$

$a = 10$

$y = 10(x-2)^2$

$y = 10(x^2 - 4x + 4)$

$\boxed{y = 10x^2 - 40x + 40}$

Ex. 3 Algebraically determine an equation, in factored form, of the parabola that has x-intercepts $3+\sqrt{7}$ and $3-\sqrt{7}$, and that passes through the point (-5,3).

$y = a(x-r)(x-s)$

Sub in intercepts

$y = a(x-(3+\sqrt{7}))(x-(3-\sqrt{7}))$

$y = a(x-3-\sqrt{7})(x-3+\sqrt{7})$

Sub in $(-5,3)$ for (x,y)

$3 = a(-5-3-\sqrt{7})(-5-3+\sqrt{7})$

$3 = a(-8-\sqrt{7})(-8+\sqrt{7})$

$3 = a(64 - \cancel{8\sqrt{7}} + \cancel{8\sqrt{7}} - 7)$

$3 = a(57)$

$\frac{3}{57} = a$

$\frac{1}{19} = a$

$\therefore f(x) = \frac{1}{19}(x-3-\sqrt{7})(x-3+\sqrt{7})$

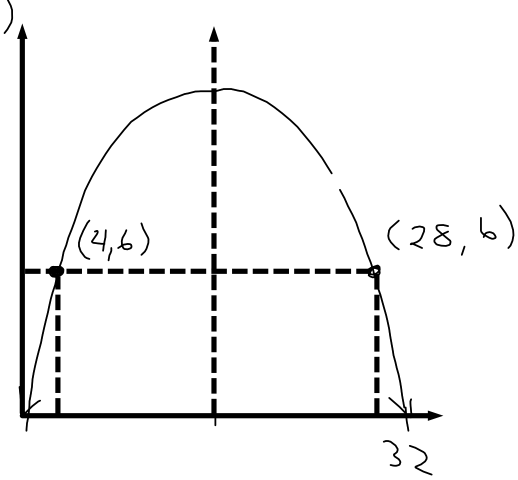
Ex. 4 The parabolic opening to a tunnel is 32 m wide measured from side to side along the ground. At points 4 m from each side, the tunnel entrance is 6 m high.

a) Sketch a diagram of the given information.

b) Determine the equation of the function that models the opening to the tunnel.

c) Find the maximum height of the tunnel, to the nearest tenth.

a)



b) Given the zeros! Use factored form

$y = a(x-r)(x-s)$

$y = a(x)(x-32)$

$y = ax(x-32)$

Sub in (4,6)

$6 = a(4)(4-32)$

$6 = a(-112)$

$-\frac{6}{112} = a$

$-\frac{3}{56} = a$ $\quad \therefore f(x) = -\frac{3}{56}x(x-32)$

c) Axis of symm:

$x = \frac{0+32}{2}$

$= 16$

$f(16) = -\frac{3}{56}(16)(16-32)$

$= -\frac{3}{56}(16)(-16)$

$\doteq 13.7$

$\therefore$ The max height of the tunnel is approx. 13.7 m

HOMEWORK

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