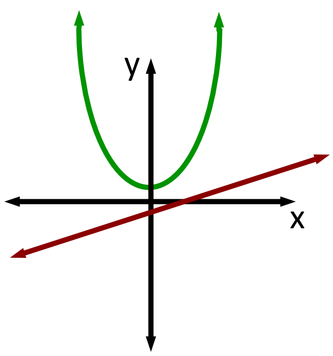
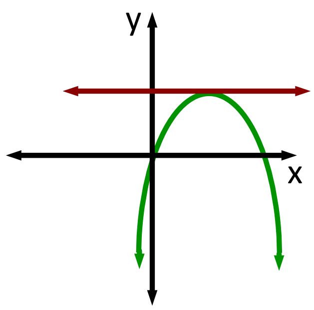
1.8 Solving Linear and Quadratic Systems

A system of equations consists of two or more equations. If the graphs in the system are linear (degree 1) and quadratic (degree 2), the system could have no solution, one solution, or two solutions.

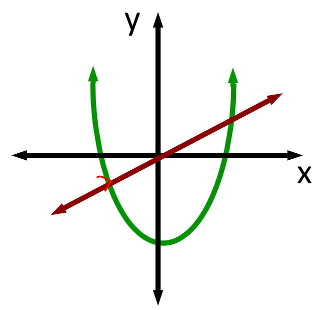
No Solution



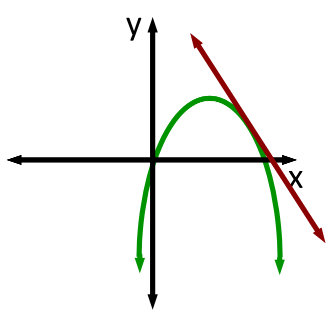
One Solution



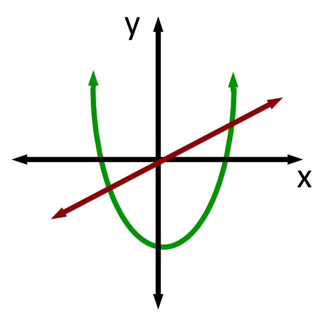
Two Solutions



Tangent - A line that intersects a curve at one point and has the same slope as the curve at that point.



Secant - A line that intersects a curve at two distinct points.



Process for solving a linear-quadratic system algebraically:

1. Isolate one variable from the linear equation.
2. Sub into the quadratic.
3. Solve for the remaining variable.
4. Sub answer(s) back into the linear equation to find the coordinate(s) of intersection, if they exist.

Ex. 1 Solve the system.

① $y = x^2 - 3$

② $2x + y = -3$

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Step 1

From ② $y = -3 - 2x$

Step 2 - Sub into ①

$-3 - 2x = x^2 - 3$

Step 3 - Solve for x

$0 = x^2 + 2x$

$= x(x+2)$

$x = 0$ $x = -2$

Step 4 Sub back into ②

$2(0) + y = -3$

$y = -3$

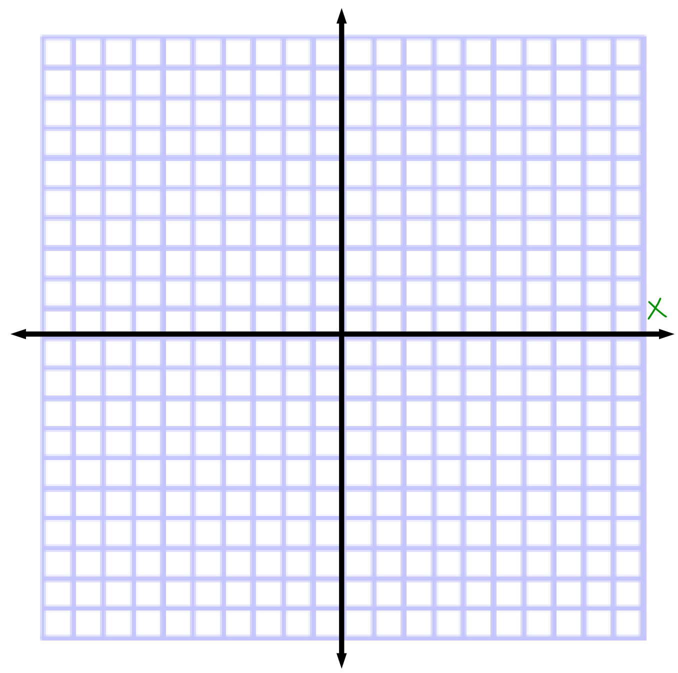
$\therefore (0, -3)$

$2(-2) + y = -3$

$y = 1$

$(-2, 1)$

Graphically



Ex. 2 Find the coordinates of the point of intersection between the parabola $y-4 = -(x + 1)^2$ and the line $y = 3x + 13$.

① $y-4=-(x+1)^2$

② $y=3x+13$

Sub ② into ①

$3x+13-4=-(x+1)^2$

$3x+9=-(x^2+2x+1)$

$0=-x^2-2x-1-3x-9$

$=-x^2-5x-10$

M 10
A -5
?

Quad!

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{5\pm\sqrt{(-5)^2-4(-1)(-10)}}{2(-1)}$

$=\frac{5\pm\sqrt{25-40}}{-2}$

Negative!

$\therefore$ No solutions

Ex. 3 If a line with a slope of 4 has one point of intersection with the quadratic function $y=\frac{1}{2}x^2 + 2x - 8$, what is the y-intercept of the line? Write the equation of the line in slope y-intercept form.

① $y=4x+b$

② $y=\frac{1}{2}x^2+2x-8$

Sub ① into ②

$$4x+b = \frac{1}{2}x^2+2x-8$$

$$0 = \frac{1}{2}x^2-2x-8-b$$

$a=\frac{1}{2}$ $b=-2$ $c=-8-b$

Since we want one solution, D must equal zero

$$D = b^2-4ac$$

$$0 = (-2)^2-4(\tfrac{1}{2})(-8-b)$$

$$= 4-2(-8-b)$$

$$= 4+16+2b$$

$$= 20+2b$$

$$-2b = 20$$

$$b=-10$$

$\therefore$ y-int is -10

Homework

p. 198 # 1a, 2ab, 3, 4, 5, 10, 11