2.1 Functions and Equivalent Expressions

In grade 12, you will learn how to graph polynomial functions and rational functions. To prepare for this work, in grade 11 you learn how to simplify rational expressions.

Ex. 1 Simplify the polynomial expression: $3(x-1)^2-(2x+1)(2x-1)$

$(x-1)(x-1)$

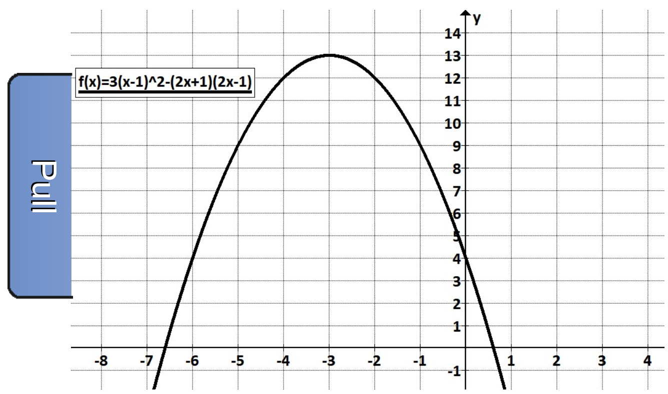
$= 3(x^2-2x+1) - (4x^2-\cancel{2x}+\cancel{2x}-1)$

$= 3x^2-6x+3 - 4x^2+1$

$= -x^2-6x+4$

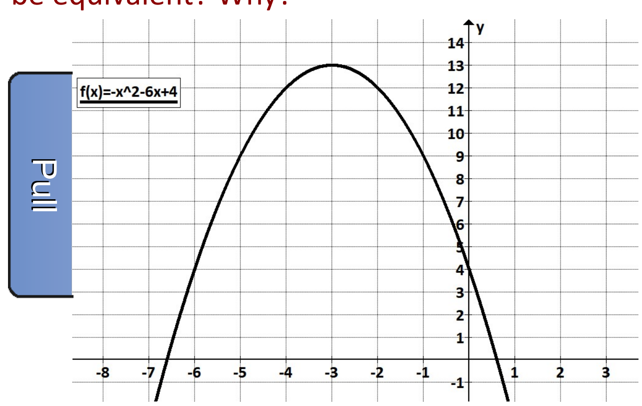
What do you think the graph of the polynomial function $f(x)=3(x-1)^2-(2x+1)(2x-1)$ will look like? Why?

- Highest degree was 2 $\therefore$ Parabola



Will the graph of the polynomial function $f(x)=-x^2-6x+4$ be equivalent? Why?

Yes!
- Same expression



Note: Talk about domain here... and whether substituting values for x is enough to show equivalency.

Need more than simply testing x

ex: $x=2$

What is a rational expression and how do we simplify them?

- Recall that a rational number is any number that can be expressed in the form $\frac{a}{b}$ where $b \neq 0$.

- Likewise, a rational expression is the quotient of two polynomial expressions $\frac{p(x)}{q(x)}$ where $q(x) \neq 0$.

$\frac{P(x)}{\emptyset}$ ← Undefined

- To simplify rational numbers,

we divide out common factors.

$$\frac{\overset{4}{\cancel{20}}}{\underset{5}{\cancel{25}}} = \frac{4}{5}$$

The same process is used to simplify rational expressions.

Ex. 2 Simplify the polynomial expression:

$$\frac{x^2-5x+6}{x^2+2x-15}$$ Factor!

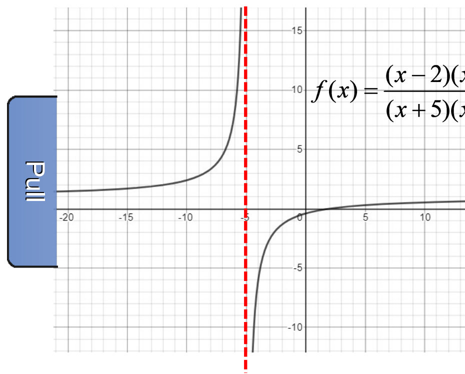
$$= \frac{(x-2)\cancel{(x-3)}}{\cancel{(x-3)}(x+5)}$$

$$\frac{\cancel{x-3}}{\cancel{x-3}} \cdot \frac{x-2}{x+5}$$

$$= \frac{x-2}{x+5}$$

$$\frac{20}{25} = \frac{5 \cdot 4}{5 \cdot 5} = \frac{\cancel{5}^{1}}{\cancel{5}} \cdot \frac{4}{5} = \frac{4}{5}$$

Let's look at the graph of this rational function:



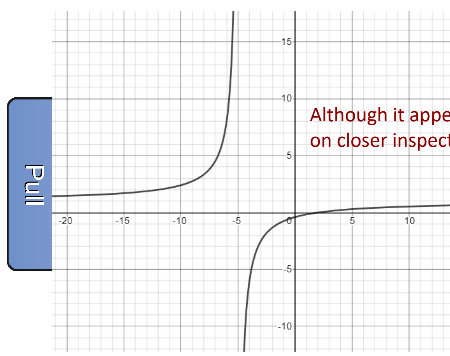
$$f(x) = \frac{(x-2)(x-3)}{(x+5)(x-3)}$$

What is happening at $x = -5$?

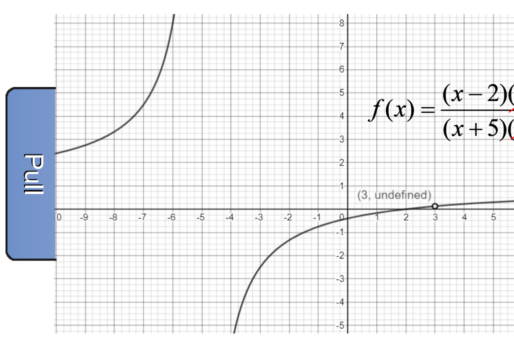
This value of *x* causes the function to be undefined and is therefore a restriction on the domain.

Asymptote

Is the graph of the rational function $f(x) = \dfrac{x-2}{x+5}$ equivalent?



Although it appears that the functions are equivalent, on closer inspection there is an important difference.



$$f(x) = \frac{(x-2)\cancel{(x-3)}}{(x+5)\cancel{(x-3)}}$$

This gap, or discontinuity in the graph, is called a hole, and is an important feature to include in graphs of rational functions. This value of x causes the function to be undefined and is therefore a restriction to the domain.

$x \neq -5, 3$

Ex. 3 Simplify each expression and determine any restrictions.

Process
1. Factor the numerator and denominator.
2. Divide out any common factors.
3. State restrictions.

a) $\dfrac{5x^2+10x}{2x^2+4x}$

$$= \left.\frac{5\cancel{x}(\cancel{x+2})}{2\cancel{x}(\cancel{x+2})}\right\}$$ LOOK HERE FOR RESTRICTIONS

$= \dfrac{5}{2}, \ x \neq -2, 0$

To state restrictions, determine the value(s) of the variable that make the denominator equal to zero.
Restrictions are placed after factoring but before simplifying.

b) $\dfrac{2x-1}{4-8x} = \dfrac{\cancel{2x-1}}{-4(\cancel{2x-1})}$

$= -\dfrac{1}{4}, \ x \neq \dfrac{1}{2}$

$2x-1=0$
$2x=1$
$x=\frac{1}{2}$

c) $\dfrac{8x^3-4x^2+6x}{2x^2}$

Can't factor

M 12
A −2

$= \dfrac{\cancel{2x}(4x^2-2x+3)}{\cancel{2x^2}}$

$= \dfrac{4x^2-2x+3}{x}, \ x \neq 0$

d) $\dfrac{x^2+x}{x^2+2x+1}$

$= \dfrac{x(\cancel{x+1})}{(x+1)(\cancel{x+1})}$

$= \dfrac{x}{x+1}, \ x \neq -1$

e) $\dfrac{2x^2+7x-15}{4x^2-9}$

M −30
A 7

$\frac{10}{2}$ $\frac{-3}{2}$

$\frac{5}{1}$

$= \dfrac{(x+5)(\cancel{2x-3})}{(\cancel{2x-3})(2x+3)}$

$= \dfrac{x+5}{2x+3}, \ x \neq \pm\dfrac{3}{2}$

Ex. 4 State the restrictions.

a) $\dfrac{4xy^3}{12x^5y^2}$

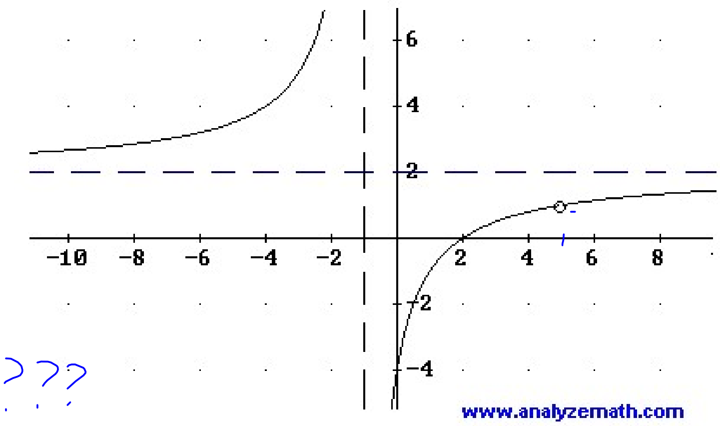
$x \neq 0$
$y \neq 0$

b) $\dfrac{1}{x^2+9}$

$x^2+9=0$
$x^2=-9$
$x=\pm\sqrt{-9}$???

$\therefore$ NO RESTRICTIONS

c)



$x \neq 5, -1$

Ex. 5 Write a rational expression ~~is~~ in one variable such that the restrictions are $x \neq \dfrac{-1}{3}, \dfrac{1}{2}$.

$x = -\frac{1}{3}$
$3x = -1$
$3x+1=0$

$x = \frac{1}{2}$
$2x = 1$
$2x-1=0$

$\dfrac{1}{(3x+1)(2x-1)}$

HOMEWORK
Pg. 113 # 2ac, 3, 4bcd, 5ad
+ Additional HW Handout Lesson 2.1