MCR3U Unit 1 Test: Functions AM
September, 2022
Curriculum Expectations: F1/F2/F3

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Name: ____________________

This section of the test is to be completed without the use of a scientific calculator.
Once handed in, you will not be able to review your work on this section.

1. **Simplify**. Write the answer in the space provided. [5]

a) $\sqrt{54} =$ $3\sqrt{6}$

b) $3\sqrt{10}(-\sqrt{10}) =$ -30

c) $-6\sqrt{11} + 3\sqrt{11} =$ $-3\sqrt{11}$

d) $\dfrac{4\sqrt{75}}{\sqrt{3}} =$ 20

e) f) $\sqrt[3]{88} =$ $2\sqrt[3]{11}$

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2. **Simplify**. Leave all answers in simplest form, rationalizing denominators where necessary. [6]

a) $3\sqrt{45} - \sqrt{32} + \sqrt{98} - \sqrt{500}$

$= 3\sqrt{9\cdot 5} - \sqrt{16\cdot 2} + \sqrt{49\cdot 2} - \sqrt{100\cdot 5}$

$= 3\cdot 3\sqrt{5} - 4\sqrt{2} + 7\sqrt{2} - 10\sqrt{5}$ ✓

$= 9\sqrt{5} - 4\sqrt{2} + 7\sqrt{2} - 10\sqrt{5}$

$= 3\sqrt{2} - \sqrt{5}$ ✓

b) $(2\sqrt{7} - 1)^2$

$= 4(7) - 4\sqrt{7} + 1$ ✓

$= 29 - 4\sqrt{7}$ ✓

c) $\dfrac{2-\sqrt{8}}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \dfrac{2\sqrt{5} - \sqrt{40}}{5}$ ✓

$= \dfrac{2\sqrt{5} - 2\sqrt{10}}{5}$ ✓

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3. **Solve** using the quadratic formula. Leave your answer in exact, simplified form. [3]

$4x^2 + 8x - 1 = 0$

$x = \dfrac{-8 \pm \sqrt{8^2 - 4(4)(-1)}}{2(4)}$ ✓

$x = \dfrac{-8 \pm \sqrt{64 + 16}}{8}$

$x = \dfrac{-8 \pm \sqrt{80}}{8}$ ✓

$x = \dfrac{-8 \pm \sqrt{16\cdot 5}}{8}$

$x = \dfrac{-8 \pm 4\sqrt{5}}{8}$ ✓

$x = \dfrac{-2 \pm \sqrt{5}}{2}$

4. Determine the **factored form equation** of the quadratic that has x-intercepts of $5 + \sqrt{3}$ and $5 - \sqrt{3}$ and passes through the point $(9, -15)$. [3]

$r = 5 + \sqrt{3}$
$s = 5 - \sqrt{3}$
$x = 9$
$f(x) = -15$

$f(x) = a(x - r)(x - s)$ ✓

$-15 = a(9 - (5 + \sqrt{3}))(9 - (5 - \sqrt{3}))$

$-15 = a(9 - 5 - \sqrt{3})(9 - 5 + \sqrt{3})$

$-15 = a(4 - \sqrt{3})(4 + \sqrt{3})$

$-15 = a(16 - 3)$ ✓

$-15 = 13a$

$\dfrac{-15}{13} = a$ ✓

$\therefore f(x) = \dfrac{-15}{13}(x - 5 - \sqrt{3})(x - 5 + \sqrt{3})$.

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Calculator use is permitted. <u>No decimals are allowed unless indicated</u>. (PM) Name: ______________

5. Determine the **domain** and **range** and indicate by circling whether the relation is a **function**, as indicated. [6]

$f(x) = \frac{1}{2}(x+5)^2 + 2$

D = $\{x \in \mathbb{R}\}$

R = $\{y \in \mathbb{R} \mid y \geq 2\}$ ✓✓

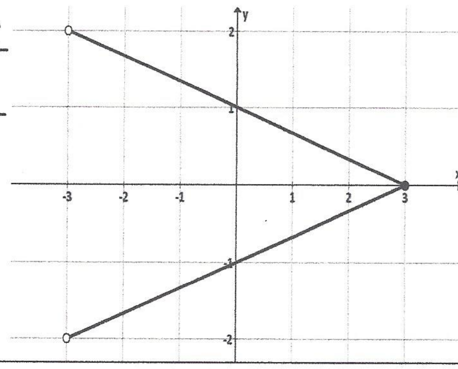
Function/Not a function?

D = $\{x \in \mathbb{R} \mid -3 < x \leq 3\}$

R = $\{y \in \mathbb{R} \mid -2 < y < 2\}$

Function/Not a function?

✓✓½



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$\{(-10,4), (-5,3), (0,0), (5,3), (10,4)\}$

many-to-one

D = $\{-10, -5, 0, 5, 10\}$ **R =** $\{0, 3, 4\}$

Function/Not a function? ✓½

6. Given $f(x) = x^2 - 3x$ and $g(x) = 5 - 2x$, **determine** and **simplify** where appropriate: [7]

a) $g(4)$

$= 5 - 2(4)$

$= -3$ ✓

b) $f(m+2)$

$= (m+2)^2 - 3(m+2)$

$= m^2 + 4m + 4 - 3m - 6$

$= m^2 + m - 2$ ✓✓

c) x when $f(x) = 10$

$10 = x^2 - 3x$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$\therefore x = 5$ or $x = -2$ ✓✓

d) $g(g(x))$

$= g(5-2x)$

$= 5 - 2(5-2x)$

$= 5 - 10 + 4x$

$= 4x - 5$ ✓✓

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7. Determine the **vertex** of $f(x) = -3x^2 + 30x + 73$ by <u>partial factoring</u>. [3]

$f(x) = -3x^2 + 30x + 73$

$= -3x(x - 10) + 73$ ✓

Symm. Pts: $(0, 73)$ $(10, 73)$

AOS: $x = \frac{0+10}{2}$ ✓

$x = 5$

$f(5) = -3(5)^2 + 30(5) + 73$ ✓

$= -75 + 150 + 73$

$= 148$

$\therefore$ The vertex is $(5, 148)$.

8. Determine the **minimum value** of $f(x) = 3x^2 - 5x - 4$ by <u>completing the square</u>. [3]

$f(x) = 3x^2 - 5x - 4$

$= 3\left(x^2 - \frac{5}{3}x\right) - 4$ ✓

$= 3\left(x^2 - \frac{5}{3}x + \frac{25}{36} - \frac{25}{36}\right) - 4$ ✓

$= 3\left(x - \frac{5}{6}\right)^2 - \frac{25}{12} - \frac{48}{12}$

$= 3\left(x - \frac{5}{6}\right)^2 - \frac{73}{12}$ ✓

$\left(\frac{-5}{3} \times \frac{1}{2}\right)^2 = \left(-\frac{5}{6}\right)^2 = \frac{25}{36}$

$\therefore$ The min value is $f(x) = -\frac{73}{12}$.

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8. **Solve** each quadratic equation <u>without the use of the quadratic formula</u>. [4]

a) $25x^2 + 40 = 70x$

$25x^2 - 70x + 40 = 0$

$5(5x^2 - 14x + 8) = 0$

$5(5x - 4)(x - 2) = 0$

$\therefore x = \frac{4}{5}$ or $x = 2$

M 40

A −14 ✓✓

N −4, −10

$\frac{5}{-4}, \frac{5}{-10} = \frac{1}{-2}$

b) $\frac{1}{3}(x-2)^2 - 4 = 20$

$\frac{1}{3}(x-2)^2 = 24$ ✓✓

$(x-2)^2 = 72$

$x - 2 = \pm\sqrt{72}$

$x - 2 = \pm 6\sqrt{2}$

$x = 2 \pm 6\sqrt{2}$

4

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9. **Solve** the following linear quadratic system. [4]

① $y = 3x^2 - 8x + 5$

② $x + y - 3 = 0$

In ②: $y = 3 - x$ ✓

Sub into ①: $3 - x = 3x^2 - 8x + 5$

$0 = 3x^2 - 7x + 2$

$0 = (3x - 1)(x - 2)$ ✓

$\therefore x = \frac{1}{3}$ or $x = 2$

Sub. into ②: $\frac{1}{3} + y - 3 = 0$ or $2 + y - 3 = 0$ ✓

$y = 3 - \frac{1}{3}$ $\quad$ $y = 1$

$y = \frac{8}{3}$ ✓

$\therefore$ The solutions are $\left(\frac{1}{3}, \frac{8}{3}\right)$ and $(2, 1)$.

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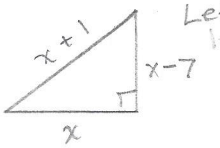
10. Show a full, <u>algebraic</u> solution for **EITHER problem A or B.** DO NOT solve both problems! [4]

Choice A: The hypotenuse of a right triangle is 1 centimeter longer than the longer leg. The shorter leg is 7 centimeters shorter than the longer leg. Determine the **measures of the three sides.**

Choice B: A company has determined that if the selling price of an item is \$90, then 1088 items will be sold. If the price is increased by \$3, then 32 less items will be sold. The cost of making the product is \$12 per item. Determine the **selling price** that will maximize the profit.



Let x be the length of the longer leg in cm.

$a^2 + b^2 = c^2$

$x^2 + (x-7)^2 = (x+1)^2$

$x^2 + x^2 - 14x + 49 = x^2 + 2x + 1$

$2x^2 - 14x + 49 = x^2 + 2x + 1$

$x^2 - 16x + 48 = 0$

$(x - 4)(x - 12) = 0$

$\therefore x = 4$ or $x = 12$

inadmissable

$\therefore$ The three sides measure 12 cm, 5 cm and 13 cm.

Let x be the # of \$3 increases to price.

Let P be the profit in \$.

$P = $ (Profit price)(Qty Sold)

$P = (90 - 12 + 3x)(1088 - 32x)$

$P = (78 + 3x)(1088 - 32x)$

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Set $P = 0$, solve for roots.

$0 = (78 + 3x)(1088 - 32x)$

$\therefore 78 + 3x = 0$ or $1088 - 32x = 0$

$3x = -78$ $\quad$ $1088 = 32x$

$x = -26$ $\quad$ $34 = x$

AOS: $x = \frac{-26 + 34}{2}$

$x = 4$

Selling price $= 90 + 3x$

$= 90 + 3(4)$

$= 102$

$\therefore$ The best selling price is \$102.

11. <u>Algebraically</u> determine the **value(s) of k** that would result in the graph of $f(x) = x^2 + (k-1)x + 1$ crossing the x-axis **twice.** [2]

$a = 1 \quad b = k - 1 \quad c = 1$

If the graph crosses the x-axis twice,

$\therefore b^2 - 4ac > 0$ ✓

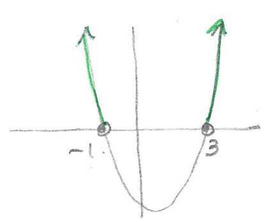
$(k-1)^2 - 4(1)(1) > 0$

$k^2 - 2k + 1 - 4 > 0$

$k^2 - 2k - 3 > 0$ ✓

$(k - 3)(k + 1) > 0$

$\therefore k < -1$ or $k > 3$



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