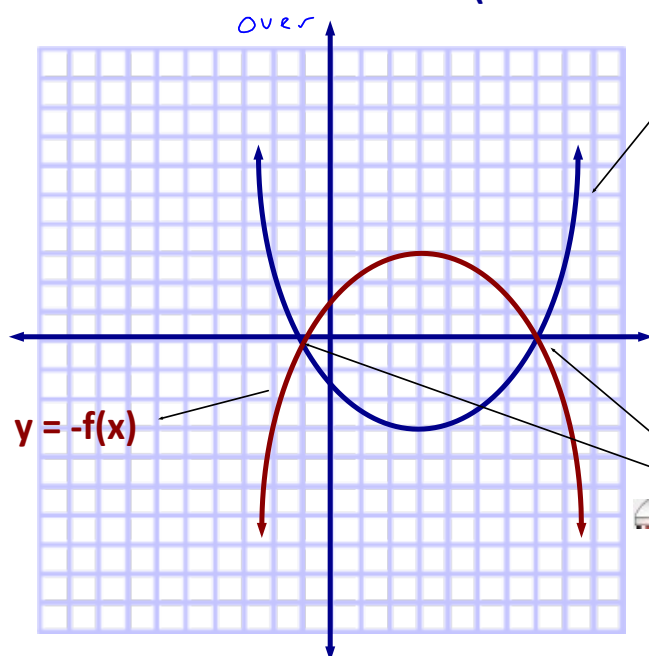## Lesson 2.4: Reflections   A reflection creates a mirror image.

A.   Reflection ~~in~~ the x-axis (vertical reflection).

over

y = f(x)

What do you notice about the y-values?

y-values are flipped

What do you notice about the x-values?

Do not change

y = -f(x)

Invariant points
are on the
x-axis.

Ex. 1:

a)   Graph $f(x) = (x - 3)^2 + 5$

b)   Find the equation of $g(x) = -f(x)$.

$$g(x) = -[(x-3)^2 + 5]$$
$$= -(x-3)^2 - 5$$

c)   Graph g(x).
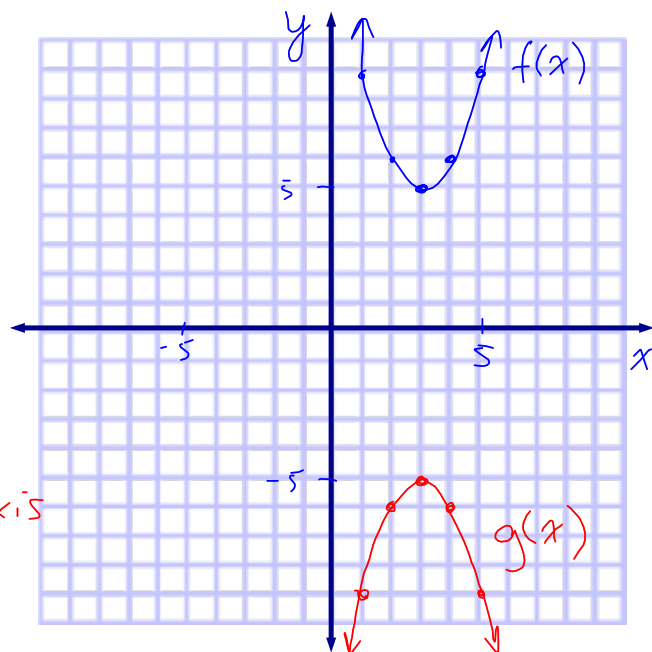
Reflected over x-axis

d)   List any invariant points.

None

f(x)

g(x)

e)   State the domain and range of f(x) and g(x).

f(x): D= $\{x \in \mathbb{R}\}$ ←SAME→ g(x): D= $\{x \in \mathbb{R}\}$

R= $\{y \in \mathbb{R} | y \geq 5\}$ ←FLIPPED→ R= $\{y \in \mathbb{R} | y \leq -5\}$

Ex. 2:

$\dfrac{\sqrt{x}}{}$

0	0
1	1
4	2
9	3

a) Graph $f(x) = \sqrt{x+2} + 3$

up 3

2 LEFT

b) Find the equation of $g(x) = -f(x)$.

$$g(x) = -\left(\sqrt{x+2} + 3\right)$$
$$= -\sqrt{x+2} - 3$$

c) Graph $g(x)$.

Reflection over x-axis

d) List any invariant points.
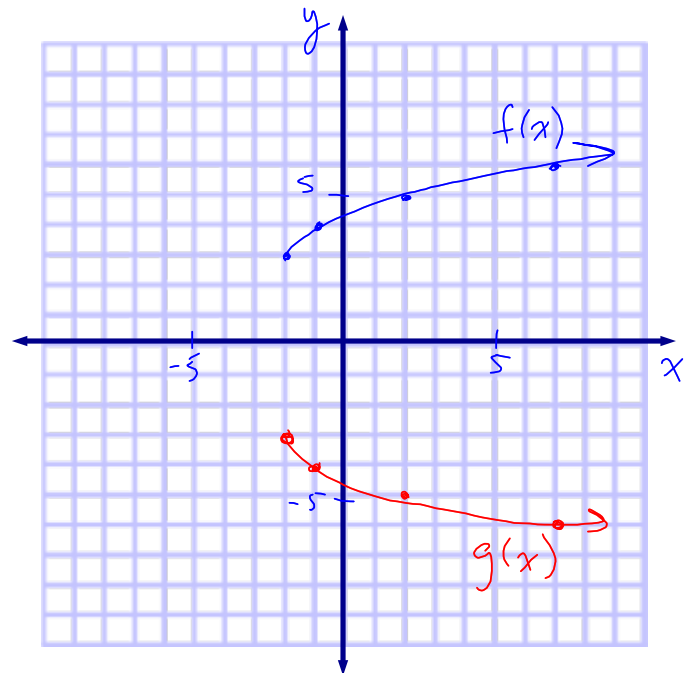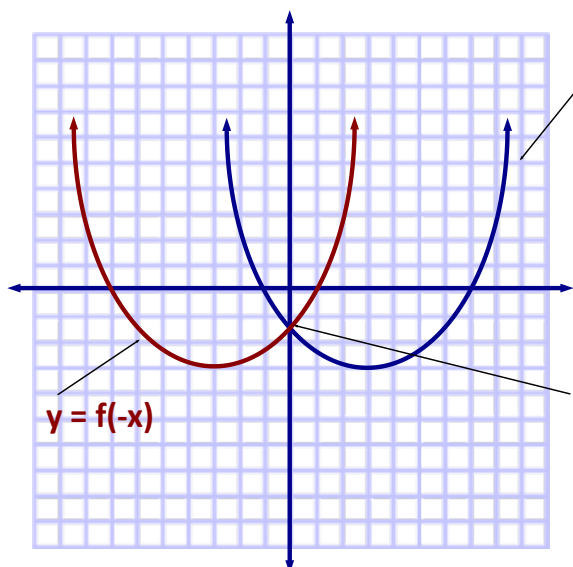
None

e) State the domain and range of $f(x)$ and $g(x)$.

$f(x)$: D = $\{x \in \mathbb{R} \mid x \geq -2\}$ $g(x)$: D = $\{x \in \mathbb{R} \mid x \geq -2\}$

R = $\{y \in \mathbb{R} \mid y \geq 3\}$ R = $\{y \in \mathbb{R} \mid y \leq -3\}$

B.   Reflection in the y-axis (horizontal reflection).

$y = f(x)$

What do you notice about the x-values?

changed (flipped)

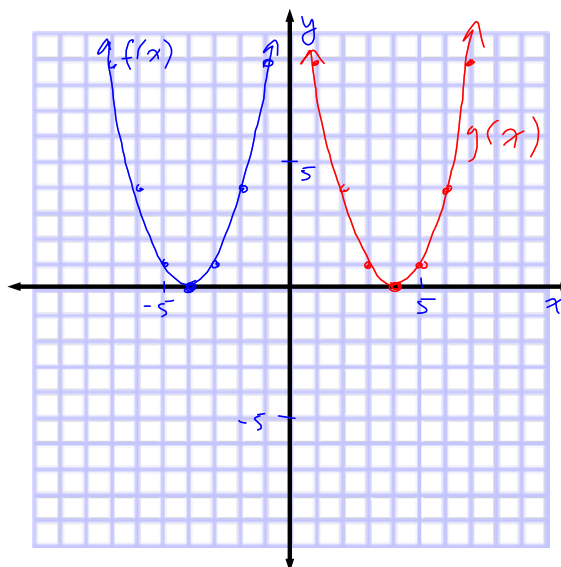What do you notice about the y-values?

stayed the same

$y = f(-x)$

Invariant points
are on the
y-axis.

Ex. 3:

a)   Graph $f(x) = (x + 4)^2$

Replace "x" with "-x".

b)   Find the equation of $g(x) = f(-x)$.

$$g(x) = (-x + 4)^2$$
$$= [(-1)(x - 4)]^2$$
$$= (-1)^2 (x - 4)^2$$
$$= (x - 4)^2$$

$f(x)$

$g(x)$

c)   Graph g(x).

Reflected over y-axis

d)   List any invariant points.

Anything on y-axis!   Set $x = 0$

$f(0) = (0 + 4)^2$
$= 16$

$\therefore (0, 16)$

e)   State the domain and range of f(x) and g(x).

f(x): D= $\{x \in \mathbb{R}\}$ g(x): D= $\{x \in \mathbb{R}\}$
      R= $\{y \in \mathbb{R} \mid y \geq 0\}$ R= $\{y \in \mathbb{R} \mid y \geq 0\}$

Ex. 4

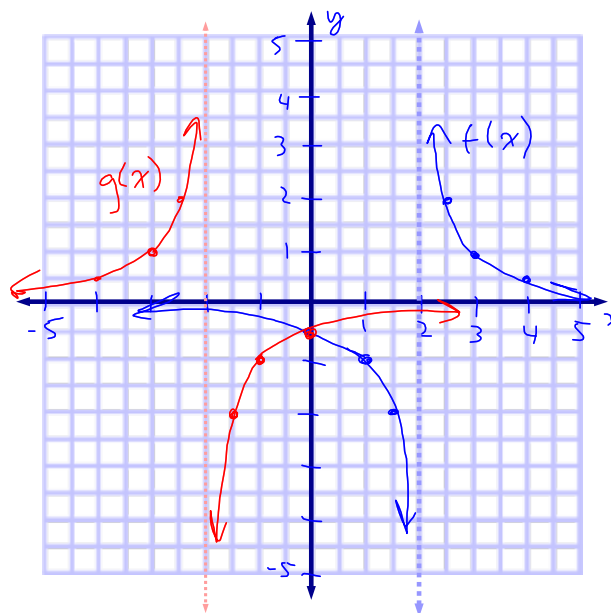a)   Graph f(x) = $\dfrac{1}{x-2}$

Base $\dfrac{1}{x}$ ↙ RIGHT 2

b)   Find the equation of g(x) = f(-x).

$$g(x) = \dfrac{1}{-x-2}$$

$$= \dfrac{1}{-1(x+2)}$$

Vert. Refl. $= -\dfrac{1}{x+2}$ LEFT 2

c)   Graph g(x).

d)   List any invariant points.

y-ints! $f(0) = -\dfrac{1}{0+2}$ ∴ $\left(0, -\dfrac{1}{2}\right)$

$= -\dfrac{1}{2}$

e)   State the domain and range of f(x) and g(x).

f(x): D= $\{x \in \mathbb{R} \mid x \neq 2\}$ g(x): D= $\{x \in \mathbb{R} \mid x \neq -2\}$
R= $\{y \in \mathbb{R} \mid y \neq 0\}$ R= $\{y \in \mathbb{R} \mid y \neq 0\}$

Summary

If y = f(x), then:

y = - f(x) represents a reflection in over __x-axis__ .
Under a reflection in the __x__-axis, the point (x, y) ⇒ (__x__ , __-y__).
Invariant points are located __x-ints__ .

y = f(-x) represents a reflection in __y-axis__ .
Under a reflection in the __y__ -axis, the point (x, y) ⇒ (__-x__ , __y__).
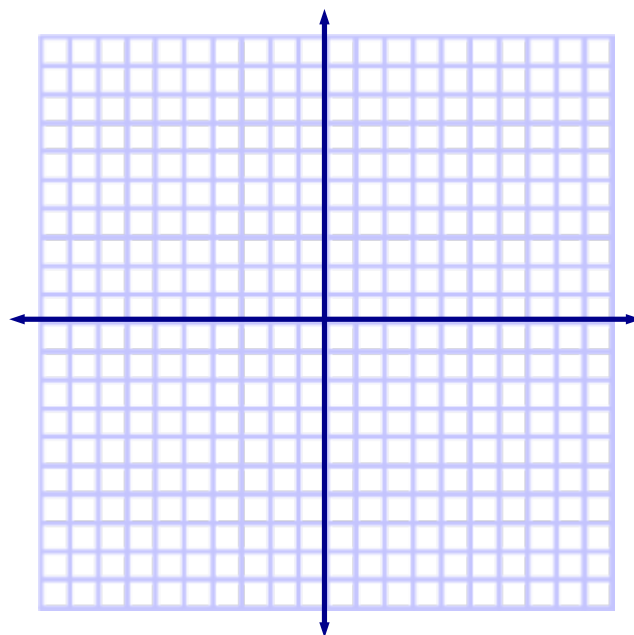Invariant points are located __y-ints__ .

y = - f(-x) represents a reflection in __x-axis__ and
__y-axis__ . The order of these reflections does not
matter. Under these reflections, the point (x, y) ⇒ (__-x__, __-y__).

Ex. 5:

Given f(x) $= |x - 3|$

a)   Graph f(x).

b)   Find the equation of g(x) = - f(-x).
      Graph g(x).

Homework
Handout Sheet 2.4
Extra Practice 2.4