Lesson 2.5: Stretches/Compressions of Functions Desmos

Part A: Vertical Stretches & Compressions $g(x) = af(x)$

$g(x) = af(x)$ is the graph of $f(x)$ that has been vertically stretched by a factor of "a".

If $a > 1$, then the graph is vertically stretched.

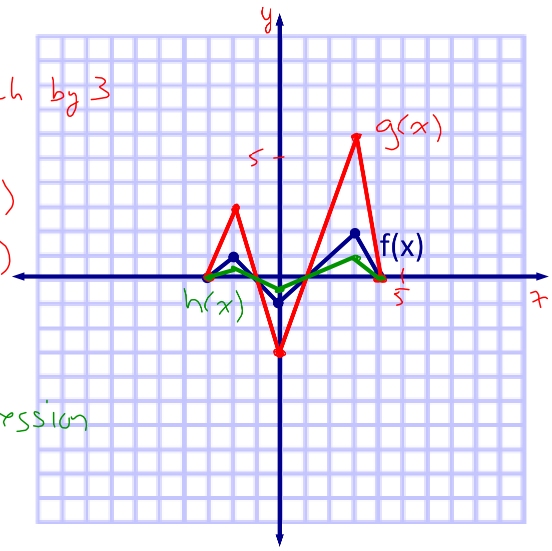
If $0 < a < 1$, then the graph is vertically compressed.

Ex. 1: Given $f(x)$ as shown, graph:

a) $g(x) = 3f(x)$ Vertical Stretch by 3

(x,y) -> (x , 3y)

(-2,1) ⇒ (-2,3)
(-3,0) → (-3,0)
(0,-1) ⇒ (0,-3)
(3,2) → (3,6)
(4,0) → (4,0)



b) $h(x) = \frac{1}{2}f(x)$ Vertical Compression by 2

(x,y) -> (x , $\frac{1}{2}$y)

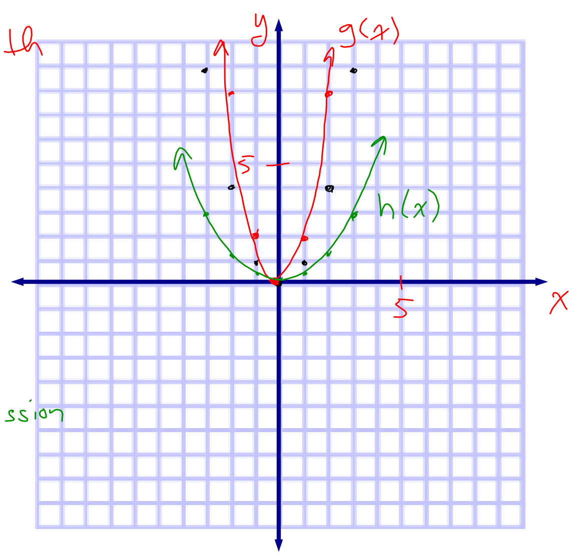
Which points are invariant? ⇒ Points lying on the x-axis (y-coord = 0)

Ex. 2: Given $f(x) = x^2$ write equations to represent g(x) and h(x) and graph :

a) $g(x) = 2f(x)$ Vertical Stretch by 2

(x,y) -> (x , 2y)

(0,0) ⇒ (0,0)
(1,1) ⇒ (1, 2)
(2,4) ⇒ (2,8)
(-1,1) ⇒ (-1,2)
(-2,4) ⇒ (-2,8)



b) $\cancel{g}h(x) = \frac{1}{3}f(x)$ Vertical Compression by 3

(x,y) -> (x , $\frac{1}{3}$y)

(1,1) → (1, $\frac{1}{3}$)
(2,4) → (2, $\frac{4}{3}$)
(3,9) → (3, $\frac{3}{3}$)

What do you notice about the domain and range? ⇒ The domain is not affected by a vertical transformation. The range is affected.

Part B: Horizontal Stretches & Compressions $g(x)=f(kx)$

Note: Use Desmos again.

$g(x)=f(kx)$ is the graph of $f(x)$ that has been horizontally stretched by a factor of "$\frac{1}{k}$".

If $k>1$, then the graph is horizontally compressed.

If $0<k<1$, then the graph is horizontally stretched.

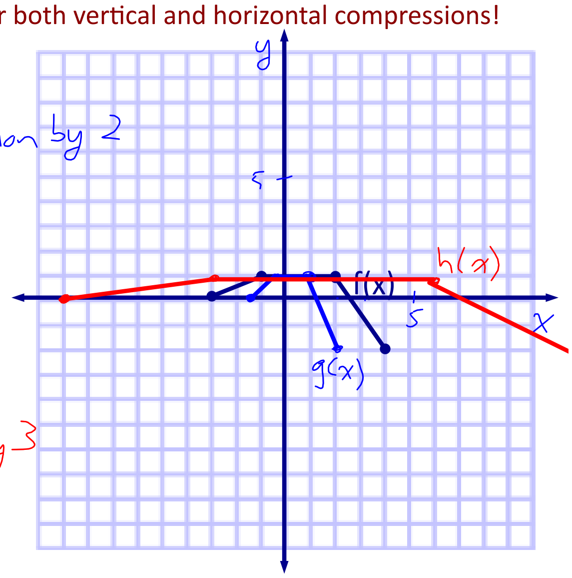
Note: k does the opposite of what you naturally think since it is inside the function.
Note: Textbook uses incorrect terminology for both vertical and horizontal compressions!

Ex. 3: Given $f(x)$, graph:

a) $g(x)=f(2x)$ Horz. Compression by 2

(x,y) -> $(\frac{1}{2}x, y)$

$(-3,0) \to (-\frac{3}{2}, 0)$
$(-1,1) \to (-\frac{1}{2}, 1)$
$(2,1) \to (1,1)$
$(4,-2) \to (2,-2)$



b) $h(x)=f\left(\frac{1}{3}x\right)$ Horz. Stretch by 3

(x,y) -> $(3x, y)$

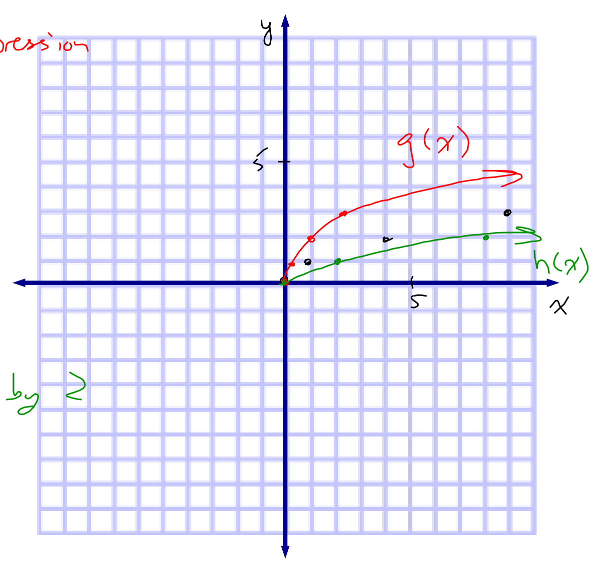
Which points are invariant? ⇒ Points lying on the y-axis (x-coord = 0)

Ex. 4: Given $f(x)=\sqrt{x}$ write equations to represent g(x) and h(x) and graph :

a) $g(x)=f(4x)$ Horizontal Compression by 4

(x,y) -> $(\frac{1}{4}x, y)$

$(0,0) \to (0,0)$
$(1,1) \to (\frac{1}{4}, 1)$
$(4,2) \to (1,2)$
$(9,3) \to (\frac{9}{4}, 3)$



b) $h(x)=f\left(\frac{1}{2}x\right)$ Horz. Stretch by 2

(x,y) -> $(2x, y)$

What do you notice about the domain and range? ⇒ The range is not affected by a horizontal transformation. The domain is affected.

Part C: Combining Horizontal & Vertical Stretches & Compressions

Ex. 5: Given $f(x) = |x|$:

a) Write an equation to represent $g(x) = 2f(2x)$.

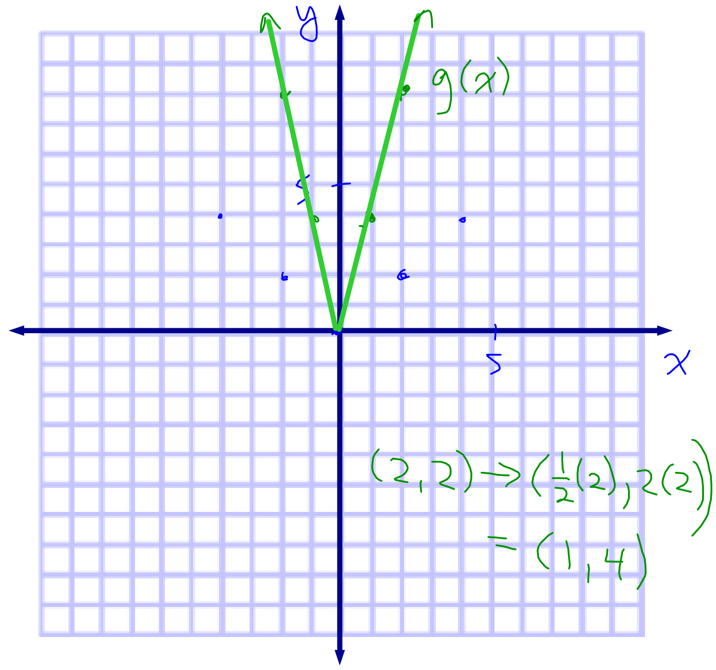
$g(x) = 2|2x|$

b) Describe the transformations.

1. Vertical Stretch by 2
2. Horiz. Compression by 2

c) Graph $g(x) = 2f(2x)$.

$(x, y) \to (\frac{1}{2}x, 2y)$



d) State the domain and range.

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq 0\}$

Ex. 6: Given that f(x) = $(2x)^2$ is a parabola that has been horizontally compressed by a factor of 2 , can you describe a different transformation that would give the SAME graph?

$f(x) = (2x)^2$

$= 4x^2$

Vertical Stretch by 4

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Homework

p. 58 # 3, 4ab, 5ab ,6ab ,7ab
Use the same two graphs!

10b, 12

Extra Practice 2.5