Lesson 2.6A: Combinations of Transformations

Summary of Transformations

If y = f(x), then describe the effect of a, k, d and c.

d represents horizontal translation (shift left/right)
if d < 0 left "d" units
if d > 0 right "d" units

c represents vertical translation (shift up/down)
if c < 0 down "c" units
if c > 0 up "c" units

$$y = a\, f[\, k\,(x - d)\,] + c$$

a represents vertical stretch/compression and/or reflection
ex: $a = \frac{1}{2}$ is compression by 2
if 0 < a < 1 vertical compression by "$\frac{1}{a}$"
if a > 1 vertical stretch by "a"
if a < 0 reflection in the x-axis

k represents horizontal stretch/compression and/or reflection
if 0 < k < 1 horiz. stretch of "$\frac{1}{k}$"
if k > 1 horiz. compression of "k"
if k < 0 reflection in the y-axis

Order of Transformations

1st: Transformations that are multiplied/divided
- Stretching /Compressing
- Reflecting

2nd: Transformations that are added/subtracted.
- Translations

Ex. 1: Describe the following transformations on a function y = f(x).

Function	Transformations to apply first	Transformations to apply second
y = -2 f(x - 3) + 1	① • Reflection in the x-axis ② • Vertical stretch of 2	③ • Horizontal translation to the right 3 units • Vertical translation up 1 unit
y = f(3(x + 2)) - 5	① • Horizontal compression by 3	② • Horizontal translation to the left 2 units • Vertical translation down 5 units
y = -5 f(-2(x - 4)) - 1	① • Reflection in the x-axis ② • Vertical stretch of 5 ③ • Reflection in the y-axis ④ • Horizontal compression by 2	⑤ • Horizontal translation to the right 4 units • Vertical translation down 1 unit

If the transformed equation is not given in standard form, you will have to factor in order to determine the correct order of the transformations.

$$y = a\, f[\, k\,(x - d)\,] + c$$

Ex. 2: Identify the transformations that have been applied to each of the following base functions.

a) $y = f(3x + 6)$

$= f[3(x + 2)]$

Factor out the coefficient of the x

① Horizontal compression by a factor of 3

② shift left 2

b) $y = \sqrt{-x + 5}$

$= \sqrt{-(x - 5)}$

① reflection in y-axis

② shift right 5 units

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$$y = a\,f[\,k\,(x - d)\,] + c$$

$$(x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$$

Ex. 3: Given the graph of f(x), transform the function as indicated.

g(x) = - f(2x + 6) - 2

$= -f[2(x+3)] - 2$ (② ① ③ ④)

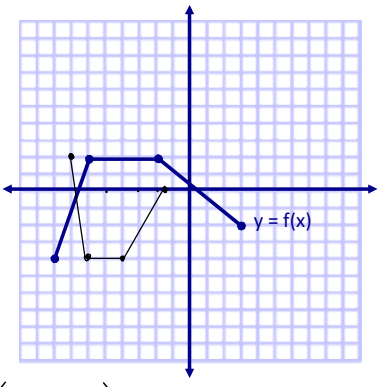
$(x,y) \Rightarrow \left(\frac{x}{2} - 3, -y - 2\right)$

① Horz. Compression by 2

② Reflection in x-axis

③ Shift left 3

④ Shift down 2



$(-2, 2) \rightarrow (-4, -4)$

$(-8, -4) \rightarrow (-7, 2)$

$(-6, 2) \rightarrow (-6, -4)$

$(3, -2) \rightarrow \left(-\frac{3}{2}, 10\right)$

Ex. 4: Write the equation of the transformed function using the given base function and transformations.

a) $f(x) = x^2$, reflected in the x-axis, vertical stretch by 3, left 6, down 2.

① $-x^2$ (② ③)

① $f(x) = -x^2$

② $f(x) = -3x^2$

③ $f(x) = -3(x+6)^2 - 2$

b) $f(x) = \frac{1}{x}$, horizontal stretch by 2, reflection in the y-axis, right 7, up 3. (① ② ③)

① $f(x) = \frac{1}{\frac{1}{2}x}$

② $f(x) = \frac{1}{-\frac{1}{2}x}$

③ $f(x) = \frac{1}{-\frac{1}{2}(x-7)} + 3$

Simplifies to

$f(x) = \frac{-2}{x-7} + 3$

c) $f(x) = \sqrt{x}$, horizontal compression by 3 (①), reflection (②) in the x-axis, ③ reflection in the y-axis, left 6, down 2. (④)

① $f(x) = \sqrt{3x}$

② $f(x) = -\sqrt{3x}$

③ $f(x) = -\sqrt{-3x}$

④ $f(x) = -\sqrt{-3(x+6)} - 2$

Ex. 5: Given f(x), write the corresponding equation and graph the transformed function.

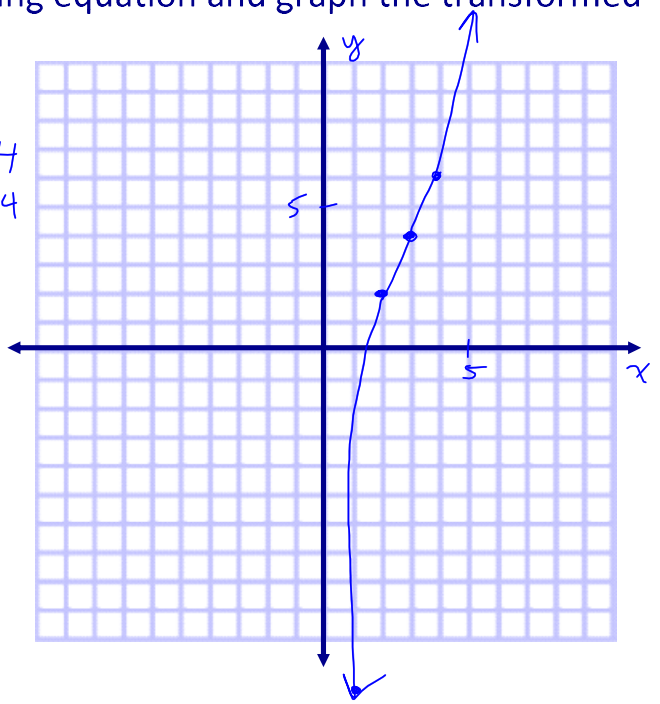
a) $f(x) = x^3$, graph $g(x) = -2f(3 - x) + 4$

$= -2f(-x+3) + 4$

$= -2f[-(x-3)] + 4$

$(x, y) \rightarrow (-x+3, -2y+4)$

$(0,0) \rightarrow (3,4)$
$(1,1) \rightarrow (2,2)$
$(2,8) \rightarrow (1,-12)$
$(-1,-1) \rightarrow (4,6)$
$(-2,-8) \rightarrow (5,20)$

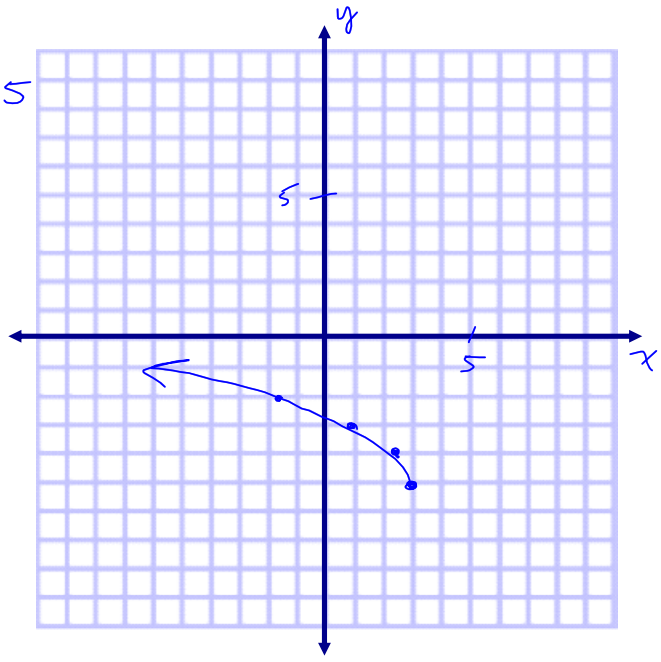


b) $f(x) = \sqrt{x}$, graph $g(x) = f(-2x + 6) - 5$

$= f[-2(x-3)] - 5$

$(x, y) \rightarrow (-\frac{x}{2}+3, y-5)$

$(0,0) \rightarrow (3,-5)$
$(1,1) \rightarrow (\frac{5}{2},-4)$
$(4,2) \rightarrow (1,-3)$
$(9,3) \rightarrow (-\frac{3}{2},-2)$



c) $f(x) = \frac{1}{x}$, graph $g(x) = \frac{1}{2}f(-x + 3) - 4$

$= \frac{1}{2}f\left[-(x-3)\right] - 4$

$(x, y) \rightarrow \left(-x+3, \frac{1}{2}y - 4\right)$

$\left(\frac{1}{2}, 2\right)$

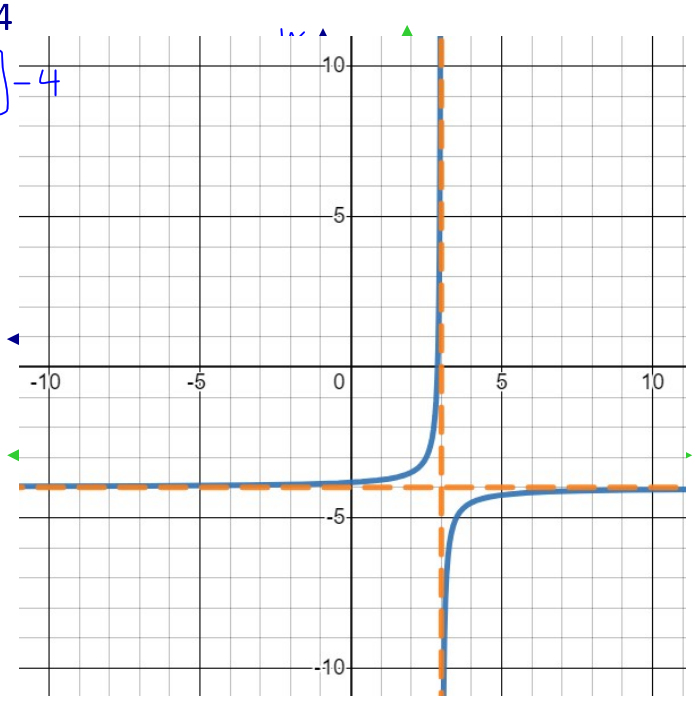
$(1, 1)$

$\left(2, \frac{1}{2}\right)$

$\left(-\frac{1}{2}, -2\right)$

$(-1, -1)$

$\left(-2, -\frac{1}{2}\right)$



Homework
p.129 #C2, 1ace, 2cde, 3abc, 4bcd, 5cdf, 7ac, 11, 13b

Extra Practice 2.6A