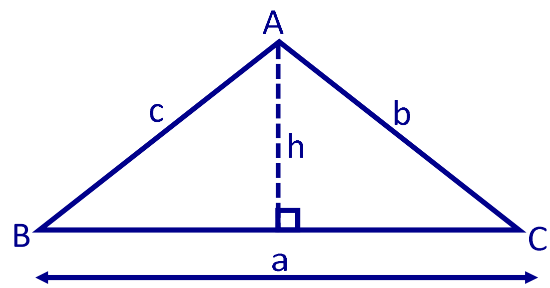
Lesson 4.4A Problems in Two Dimensions Day 1

Development of the Sine Law:

Consider ΔABC (no 90° angle).

Construct an altitude from A.

There are now 2 right triangles.

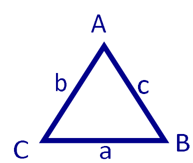


STEPS:

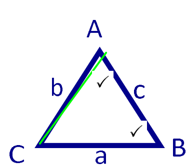
1. Write equations for sin B and sin C. ➡ $\sin B = \frac{h}{c}$ $\sin C = \frac{h}{b}$
2. Solve each equation for h. ➡ $c\sin B = h$ $b\sin C = h$
3. Since both equations = h, they must equal each other. ➡ $\therefore\ c\sin B = b\sin C$
4. Divide both sides by b and c. ➡ $\frac{\sin B}{b} = \frac{\sin C}{c}$

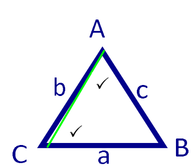
The Sine Law

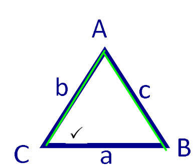
In ΔABC,



$$\frac{a}{sinA} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad or \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

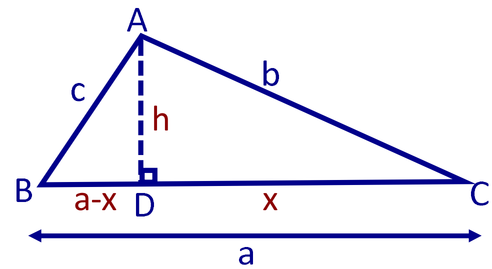






Development of the Cosine Law:

- consider ΔABC (no 90°)
- construct an altitude from A
- notice that there are now 2 right triangles



In ΔADC:

- Write the Pythagorean theorem

$$b^2 = x^2 + h^2$$
$$\therefore h^2 = b^2 - x^2$$

- Write the cosine ratio for C

$$\cos C = \frac{x}{b}$$
$$x = b \cdot \cos C$$

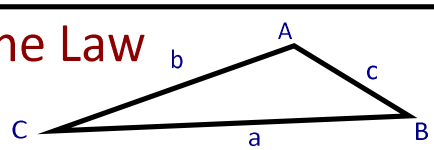
In ΔABD:

- Write the Pythagorean theorem
- Expand and simplify
- Substitute from ΔADC

$$c^2 = h^2 + (a - x)^2$$
$$c^2 = h^2 + a^2 - 2ax + x^2$$
$$c^2 = (b^2 - x^2) + a^2 - 2ax + x^2$$
$$c^2 = b^2 + a^2 - 2ax$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

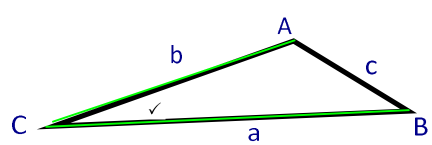
The Cosine Law

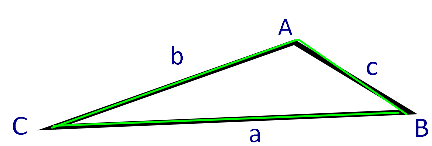
In ΔABC,

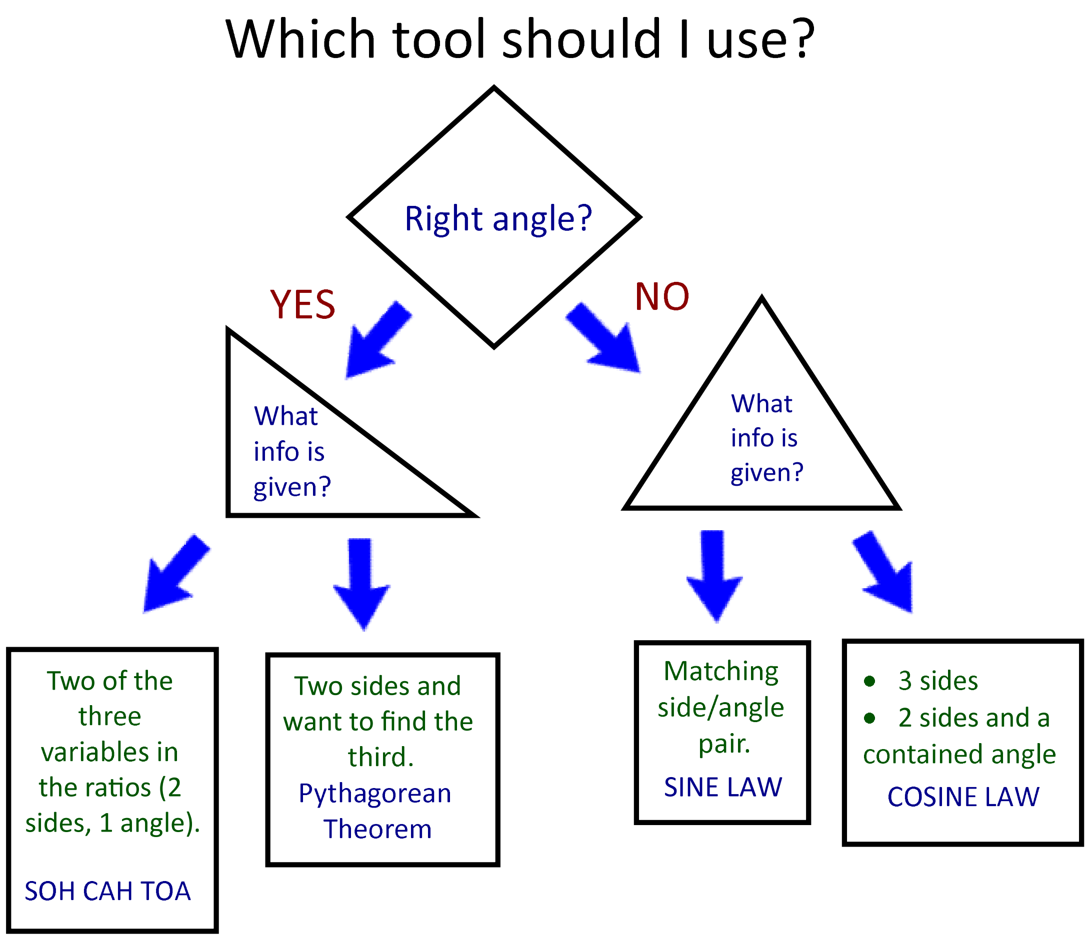


$c^2 = a^2 + b^2 - 2ab \cos C$ rearrange --> $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

(Used when finding a side). (Used when finding an angle).

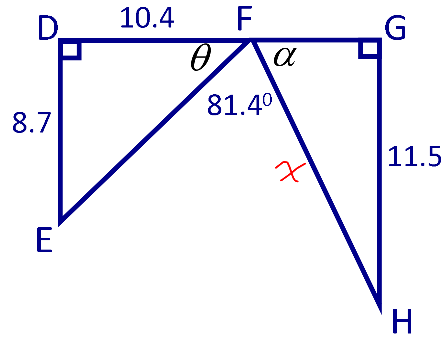





Which tool should I use?
Right angle?
YES
NO
What info is given?
What info is given?
Two of the three variables in the ratios (2 sides, 1 angle).
SOH CAH TOA
Two sides and want to find the third.
Pythagorean Theorem
Matching side/angle pair.
SINE LAW
• 3 sides
• 2 sides and a contained angle
COSINE LAW

Ex. 1 Find the length of FH.

Right Triangles! SOHCAHTOA



Step by step to get the info you need.

- Find θ
- Find α
- Find x

θ?

$$\tan\theta = \frac{8.7}{10.4}$$

$$\theta \doteq 40°$$

α?

$$\alpha = 180° - 40° - 81.4°$$
$$= 58.6°$$

x?

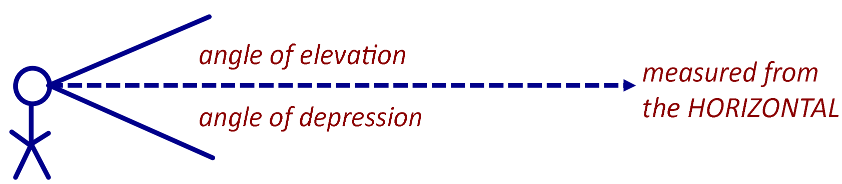
$$\sin\alpha = \frac{11.5}{x}$$

$$\sin 58.6° = \frac{11.5}{x}$$

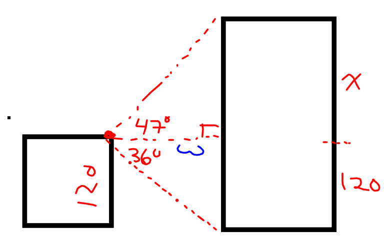
$$x = \frac{11.5}{\sin 58.6}$$

$$\doteq 13.5$$

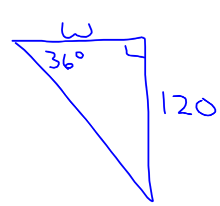
∴ The length of FH is 13.5



Ex. 2 From the top of a 120 m tall building, the angle of elevation to the top of another building is 47° and the angle of depression to the bottom of the same building is 36°. How high is the second building?



①

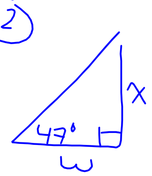


$$\tan 36° = \frac{120}{w}$$

$$w = \frac{120}{\tan 36°}$$

$$\doteq 165.2$$

②



$$\tan 47° = \frac{x}{165.2}$$

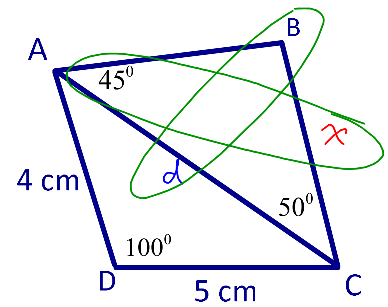
$$165.2 \cdot \tan 47° = x$$

$$x \doteq 177.2$$

∴ Height is 177.2 + 120

= 297.2 m

Ex. 3 Find the length of BC to one decimal place.



Cosine Law for AC
Sum of angles for B
Sine law for x

AC?

$d^2 = c^2 + a^2 - 2ca\cos D$

$= 4^2 + 5^2 - 2(4)(5)\cdot\cos 100^\circ$

$\doteq 47.9$

$d \doteq 6.9$

B?

$\angle B = 180 - 45 - 50$

$= 85^\circ$

x? Sine law

$\frac{a}{\sin A} = \frac{b}{\sin B}$

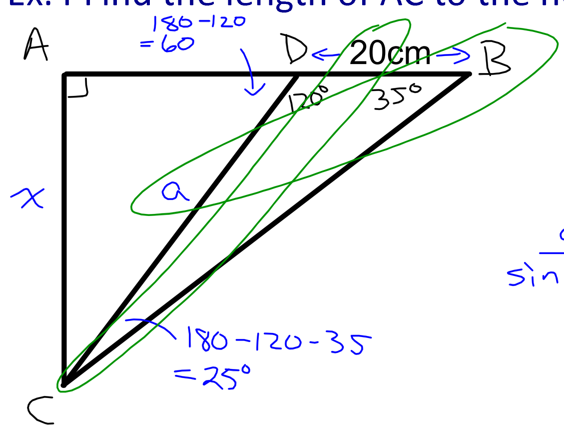
$\frac{x}{\sin 45^\circ} = \frac{6.9}{\sin 85^\circ}$

$x = \sin 45 \cdot \frac{6.9}{\sin 85}$

$= 4.9$

∴ Length of BC is 4.9cm

Ex.4 Find the length of AC to the nearest tenth.



- Find ∠DCB
- Sine Law for a
- Find ∠ADC
- Use SOH for x

a?

$\frac{a}{\sin 35^\circ} = \frac{20}{\sin 25^\circ}$

$a = \sin 35^\circ \cdot \frac{20}{\sin 25^\circ}$

$\doteq 27.14$

x?

$\sin 60^\circ = \frac{x}{27.14}$

$x \doteq 23.5$

∴ Length of AC is 23.5cm

Homework
pg. 254 #2ab, 5, 7, 10-12

Answer for 11.b should be 22.8^0 west of south.